BLOTT MATTHEWS A2A CHALLENGE SOLUTION BY INTERPORT 20

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Get to any
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INTRODUCTION TO THE TEAM

ROB MACLENNAN -MANAGING DIRECTOR

My primary role has been to organise and oversee the running of the project - delegating tasks tailored to the preferences of each individual, ensuring smooth coordination between all members as well as setting deadlines and making sure they're met. As part of this, I also collated the important fundamental aspects of our system from across the various areas to write the overall description. Alongside my leadership role, I have made significant contributions across various departments. I particularly enjoyed doing the station designs, in which I had to find a solution whereby the tough logistical constraints of departures every 5 minutes could be realistically solved and integrate that into the varying needs of each separate station. As well as this I made the interactive Google Map route and researched and developed the basis for the timings of the construction of the network.

The main reason I wanted to take on the A2A challenge was because I love problem solving, and the project offered an exciting and complex challenge, which could conceivably take place in the real world. Through the process, I have significantly developed my leadership and organisational skills, as well as learning some really interesting physics. In my free time, I enjoy fencing for Great Britain internationally at U20 level, and recording and gigging with my band.

BEN WILSON – ACCOUNTANT, MINER AND SECRETARY

I've done work in three major sections. The first of these was tunnelling, a section which initially posed a significant challenge due to pressure considerations. The next was train kinematics and logistics; looking into how fast the trains could go, helping with other maths sections, and deciding on the construction and implementation schedule of the overall project. This linked into the final thing I worked on, which was finance. I estimated and calculated costs for the project, as well as helping with the maths in other sections.

I also helped made sure tasks were done on time (or attempted to), and collected everyone's individual sections into this final document.

I'm a big fan of all things sciencey, in particular physics and maths. The main reason I decided to join the group for this engineering challenge was that I didn't know how such large-scale projects are planned and brought into existence, and thought going through the process myself would be both interesting and useful. I've certainly learnt a lot about group organisation and project management from this experience.

JAMES HOGGE - CIVIL ENGINEER

My work in this task has been primarily on the track and train designs and the technologies involved. This has included running magnetic and airflow simulations for the trains to assist in making design decisions. A large part of my time has been spent researching into similar engineering projects from which we can gather information as well as trying to come to terms with electromagnetic systems and how to model them with maths. While some research took me well out of my depth, it has all been interesting and I think it is safe to say that I have gained from my participation.

My initial interest for the A2A challenge has come from my love of the sciences and maths, all of which I am studying for A level. I also have a strong interest in computing and technology so a chance to help design a transport system thoroughly appealed to me. I have also closely followed the Bloodhound SSC and SpaceX both of which have fuelled my enjoyment in all the STEM subjects.

PATRICK MCCUBBIN - SAFETY, RELIABILITY AND IMPLICATIONS OF THE PROJECT

I worked on the initial planning of the route, and especially how it can be integrated with existing transport networks by the location of the central hub. I then moved on to looking at the safety and reliability of the network, including breakdown response. This involved continuous dialogue with the engineering departments, ensuring appropriate redundancy in the technology. Procedures also had to be in place in case trains had to be rerouted: having initially worked on the route helped a lot with this problem. I meanwhile continued to work on the route examining issues such as greenfield/brownfield usage, power demand on the national grid and pollution - essentially some of the nitty-gritty implications of the project for both local people and national infrastructure as a whole.

Being involved in the project appealed to me because I really wanted to work in a team trying to solve a problem based firmly in the real world. I study all three sciences and further maths at A level, and want to go on to a degree in biochemistry. I am developing interests in nanotechnology and also systems biology so getting involved in a complex challenge like this rooted in physics and engineering like this was perfect for me.

JOE NASH - CARTOGRAPHER

Within the project, almost all my time was spent designing, developing and planning the route. Having decided on our original concepts, I spent a lot of time at the start of the project scouring maps of London to find all the best places that a track could be placed, and compiling these into a list for comparison. This then led to further reviewing and compiling until the final few lines remained and a final route was decided. This was then developed into a final map, and then each of the line locations was closely studied to allow for certain features of each line to be noted down. This then led to specific features such as junctions and slopes on the track to be covered, resulting in several diagrams being produced. The overall planning was then finished off, using both my own research and that of others, specifically Patrick, and the environmental aspects of the track were then covered in much the same way.

I personally have always been interested in engineering, specifically the maths-oriented side, and as such the opportunity of this project appealed to me. I greatly enjoyed improving my knowledge of the infrastructure of transport networks, as they have a slight beauty in their logic and their regular patterns in design, such as with the junctions on motorways that I studied and attempted to emulate. The project has been an extremely fulfilling process, allowing me to both explore and confirm my enjoyment in fields that I desire to enter later in life, and it has been a great chance and a brilliant challenge to work with a team of skilled people to overcome a monumental task.

PETER WANG - GRAPHICS AND ENGINEERING

My role in this project was to assist James with the designing and engineering of the trains and tracks, researching materials, working out dimensions and fitting together the little details like how the rails are going to cross each other. My job was less technical in a sense, but nonetheless it required a huge amount of trial and error, as well as attention to details, which are both challenging and exciting for me.

My research aside, I also worked on presentation and illustration, making sure the diagrams were clear and pleasing to the eye. The 3D renders in particular took three straight days of staring at a laptop screen and a software (the name of which shall not be mentioned) with the worst user friendliness I've ever seen (it's Blender), I'd rather not see a train again for the rest of my life.

KURT LEE - BAGGAGE LOGISTICS AND PUBLIC TRANSPORT ANALYSIS

Initially, I worked on analysing the current available transport systems that exist between the hubs. I looked at the different range of costs, distances and times it took to get between hubs with the current existing transportation. This was useful in giving us a general idea of the current situation and thus guided us to the final solution. Aside from this, I have mainly worked on the luggage loading system. The luggage loading system is a supporting but nonetheless integral part of the system. Developing this system required working with the station design to make sure there were no logistical conflicts.

This project appealed to me because I have always been passionate about solving real-life problems that could bring positive effects to people. I study Further maths, Physics, Economics and Politics at A-level and want to go on to a degree in Politics. In the world of politics, people are constantly faced with different problems, and I believe this project has instilled me with a creative approach of problem solving for the future. Engineering, I believe, can be closely related and in some way very similar to politics in that they both strive to solve practical problems.

OVERALL DESCRIPTION

Our solution to this year's challenge is a rail network that utilises magnetic levitation, or 'maglev' for short, to connect all 5 of the London airports together.

System Specifications	
Journey Time	Up to 20 minutes
Waiting Time	5 minutes over 60% of the time, up to 30 minutes
Max Speed	99.9 ms^{-1}
Overground Track Length	150.3 km
Underground Track Length	107.4 km
Required Loan	£26.725 billion
Time to Construct System	8 years, 3 months
Time to Pay Back Loan	15 years, 8 months after construction has completed
Annual Costs	£184.71 million (Completed System)
Annual Sales	£2,845.00 million (Completed System)
Ticket Price Range	£5.50 – £25.10 Peak; £8.20 – £37.60 Off-Peak
Annual Energy Use	2.37 Petajoules

To put the implications of our system into context, the average journey time to get between two Heathrow terminals is currently over 17 minutes. Our transportation network allows passengers to travel up to over 100km further in only 8 minutes more (including waiting time).

CHOICE OF MAGLEV

After initial research into the route we realised the longest connection (Gatwick-Luton) would be over 100km. To achieve this in 20 minutes, our trains need to have an average speed of over 350km/h.

- Due to the high speeds and high volume of traffic required to meet the given constraints, we could see that the three conceivable methods of transport were high speed rail, magnetic levitation or hyperloop.
- Despite its advantages of enormous top speeds and low maintenance costs, the hyperloop option had to be abandoned due to the vast costs associated with building hundreds of kilometres of vacuum tubes, as well as safety concerns.
- After thorough research, we decided to choose maglev over HSR because maglev trains could reach the speeds necessary to travel the longest connection in 20 minutes, with minimal noise pollution and energy costs.

CENTRAL HUB

A key feature of the network will be the central hub, located by the M40 - M25 junction. This will be the control centre for the system, allowing us to have unmanned trains, significantly reducing running costs. The almost entirely automatic system will be watched over by personnel in the central hub, who will get real-time feedback from every train, maximising safety.

- Spare maglev trains will be kept at the Central Hub, as well as a number of emergency diesel trains. Trains will be brought here for maintenance and repair.
- The Central Hub is also a station in its own right, its key location by the M40 - M25 junction making it extremely convenient to access our system.

TRAIN DESIGN

- Trains will be composed of two aerodynamically-shaped end-sections containing necessary equipment surrounding 1-3 passenger carriages with a capacity of 57 passengers.
- Alternatively, passenger carriages can hold a wheelchair user and 55 other passengers.
- The number of carriages can be changed dynamically to match demand throughout a day.
- Quartets of trains will travel back and forth between each pair of stations, each waiting 10 minutes at stations and one departing from every station every 5 minutes
- Each carriage will contain the linear synchronous motors necessary for propulsion.
- The carriages have wheels, since at low speeds they will not experience sufficient repulsion to levitate.

ROUTE

We had a number of elements to consider when designing the route: We wanted the shortest possible distance between the furthest, while avoiding towns and built-up areas, and simultaneously trying to keep as much of the route as possible overground to reduce tunnelling costs.

- Our route involves a figure of eight around London, mostly following roads and motorways. City, Heathrow and the Central Hub are directly on this loop, while tracks branch off towards Gatwick. Stansted and Luton.
- Central London is so built-up it is virtually impossible to build overground. As a result, we have several tunnels towards the City station. Several other tunnels were also required, with lengths from 1.8 km to 30.2 km.
- Short tunnels will have a single-tube, dual-track configuration, while longer tunnels will be single-track, dual tube. This is mainly due to considerations of safety and pressure waves.
- The longest route between any two stations is 106.0 km.

STATIONS

- The Heathrow, Gatwick, Luton and Stansted stations will be contained overground, beside the airports.
- The City station will be underground, beneath the airport, due to space constraints and the fact that all connecting tracks are underground.
- The Central Hub station is larger than the rest, with plenty of parking accommodation, for commuter access and holiday parking.
- The stations each contain extensive ticket and bag drop desks, information desks, toilets and cafés.
- Commuters or passengers coming from arrivals will drop off the bags at their initial station, while passengers who are catching flights are able to check in their baggage at any airport and have it transported automatically via the trains to their plane.
- Luggage not heading to planes will be collected from a carousel.

COMMUTER USE

Our transportation network will be very attractive for commuters wanting quick transport into, out of, and around London.

- The links to City in particular will be in high demand for commuters during peak periods.
- Our system will help reduce congestion and air pollution in and around central London by providing the fast, efficient alternative route into the city.

RELIABILITY

Our system has safety precautions and procedures to deal with all possible issues.

- In the case of a power loss from the National Grid, the trains will keep travelling forward on their wheels, decelerating and coming to a controlled halt.
- In the unlikely event of a breakdown, we will have diesel trains stored at the Central Hub which can run on the tracks, able to tow or shunt stranded carriages.
- One such train will travel the length of our railway on regular basis with measurement instruments, to ensure everything is in working order.
- Connections between hubs can be maintained if almost any of the track segments go down, by utilising slightly longer backup routes around other areas of the track. These will also be used before direct track segments have completed construction.
- There will be regular night-time checks of trains and annual servicing a the Central Hub.

COSTS AND TIMINGS

The project will take 8 years and 3 months to construct, with an initial loan of just over £26.725 billion.

- The bulk of revenue will be from ticket sales.
- Other, minor sources of income. will be from billboards, selling food and independent donations.
- The loan will be paid back in 23 years and 11 months.

PLANNING & ENVIRONMENT

DECIDING ON A ROUTE

OVERVIEW

One of the most critical initial areas to address at the start of the project was to plan out the route that the network would use, as this was required to work out costs, engineering difficulties and environmental effects. When deciding on a route, many variables were at play, which can be categorised into 3 main factors: minimising cost, minimising geographical disturbance, and minimising time taken for journeys. All these had to, of course, be within the physical limitations of building hundreds of kilometres of track in and around London, whilst also allowing for maximum ease when constructing it. This led to the first dilemma - to what extent should lines be overground or underground?

OVER-GROUND LINES

The route taken by overground lines is heavily dictated by pre-existing features, such as roads, towns and cities, and as such is sometimes forced to go over bridges and take less direct routes. Likewise, it must be constructed in the open, which vastly increases greenfield use, as well as noise and visual pollution. However, financially this is the cheapest option, as whole lines can be constructed simultaneously, and the resulting routes are not very much longer than direct routes.

UNDERGROUND LINES

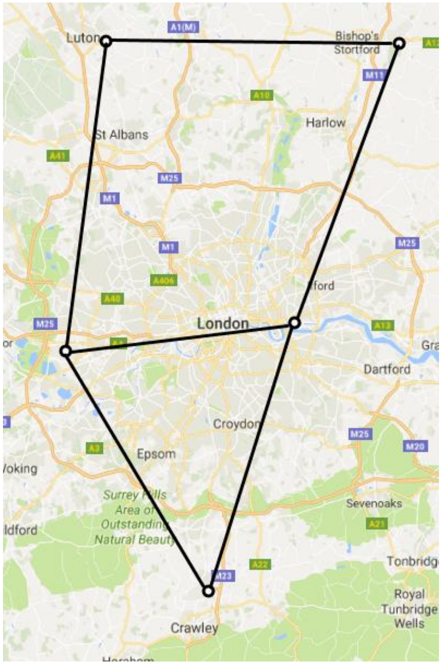
Underground lines can be mostly straight, allowing for minimal distances between airports or junctions, and they are able to lead through built up areas, including central London. Underground lines also have a very small footprint, with potentially no greenfield use and neither noise nor visual pollution. The downside is that, as well as being much more expensive, they take much longer to construct, as tunnels must be bored, meaning that tracks can only be constructed a segment at a time. Additionally, under central London, there are huge numbers of existing tunnels, which may have to be wound around at some points.

ORIGINAL CONCEPTS

Upon reviewing this information, 3 initial concepts were formed upon first inspection:

1. An entirely underground network of straight line tunnels
2. A series of curved routes linking up each airport somewhat independently
3. A large ‘S’ going through all of London, with lines to various airports leading off

Route 1



Advantages:
- Avoids the highly developed, urban parts of London
- Lowest total distance
- Very direct routes
- Extremely low Greenfield use
- Can still function if some lines are lost due to temporary shutdowns

Disadvantages:
- Tunnels are very costly
- Long construction time

Route 2

Advantages:
- Very cheap and easy to construct
- Easy access between all airports, even if several lines are lost due to temporary shutdowns
- Allows for additional pickup points from most major motorways around London and Paddington Station

Disadvantages:
- Many lines to construct
- High Greenfield use
- Very little space for construction within central London

Route 3

Advantages:
- Very cheap and easy to construct
- Allows for additional pickup points from most major motorways around London and Paddington Station
- Low total distance

Disadvantages:
- Can be brought to a halt entirely if a segment of the main line is lost temporarily
- Moderate Greenfield use
- Very little space for construction in central London

DEVELOPING THE CONCEPTS

Each of the above concepts brought its relative advantages over the other two.

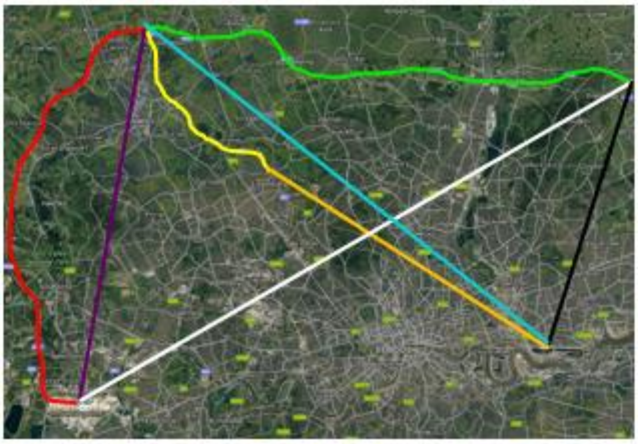
Comparing them then helped identify key features that could be taken from each of the extremes to form a hybrid solution, maintaining the advantages of each concept. For example, through the highly built-up central London, direct underground lines could be used, while through the more open areas on the outskirts, over-ground lines would be the better option. Likewise, a minimalistic design would be used to reduce cost, construction time and Greenfield use, however it would still incorporate a loop to allow for continuation of the service in the case of a temporary shutdown somewhere on the track.

FORMING A FINAL ROUTE MAP

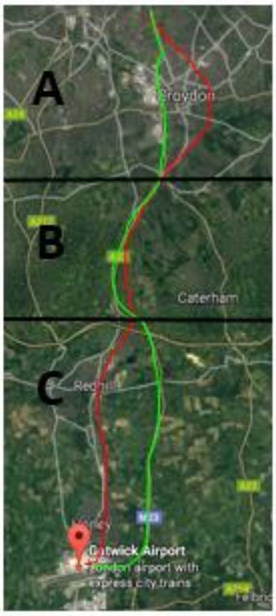
Having decided on the key details that were necessary for the route, the whole map was split up into five segments: Northern Central London, Southern Central London, the Gatwick Linkup and the Upper Airports Linkups. Each of these segments was then looked in to, in some cases split up into smaller subsections, and all possible routes that fit the criteria were detailed on the maps, with the main advantages and disadvantages of each being listed:

Northern Central London

Line	Pros	Cons
Green	Follows existing large road, fairly open space, direct	Road is not a straight line, lots of slight changes in direction
Red	Follows existing large road, very open space, mostly direct	Built up area close to Luton junction, requires some sharper turns, does change directions towards the end, comes out of the wrong side of Heathrow, requires travelling West/East at the start, not desired direction, North
Black	Direct line, short distance, completes a difficult transition	Underground
Turquoise	Direct	All underground, very long, not the best angles for transitioning into other lines
Orange/ Yellow	Follows existing road (for yellow), fairly direct, minimal underground usage (for orange)	Road is not fully straight
	Very direct, completes a difficult transition	All underground, perhaps not necessary
Purple	Direct line	Doesn't connect well with wrapping round to Stansted, all underground
Blue	Direct	All underground



Gatwick Linkup



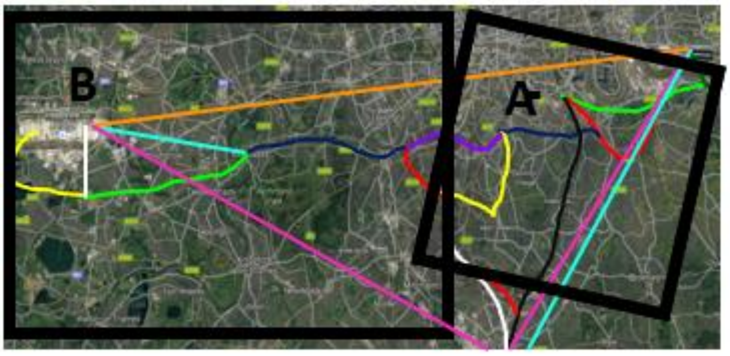
Line	Pros	Cons
Section A		
Green	Shorter and direct, short clear stretch at start	Largely built up on both sides (houses and shops)
Red	More space to develop, follows railway	Passes through Croydon train station, passes over a few roads (and under 1)
Section B		
Green	Follows on from the M23	Shrinks to small road surrounded by houses, not direct, large crowded junctions
Red	Direct, railway broadens towards the end, lots of space at start	M25 passes over railway, 150m stretch of built up area

Section C		
Green	Very few urban developments in the way, fairly open on either side, follows motorway, less disruption when developing	Longer route, sharper turning, M25 clashes
Red	Short, follows pre-existing railway	1050m stretch with built up areas on either side (+ 4 other 100-250m sections), 7 roads go over the railway line, and 10 go under

Northen Airport Linkups

Line	Pros	Cons
Green	Direct	: Must either be underground or cut through owned land (no buildings however)
Red	Follows existing road, large development space	Indirect
Black	Direct	All underground
Blue	Follows road, large development space	Slight turns in road, leaves road for short part before turning to Stansted

Southern Central London

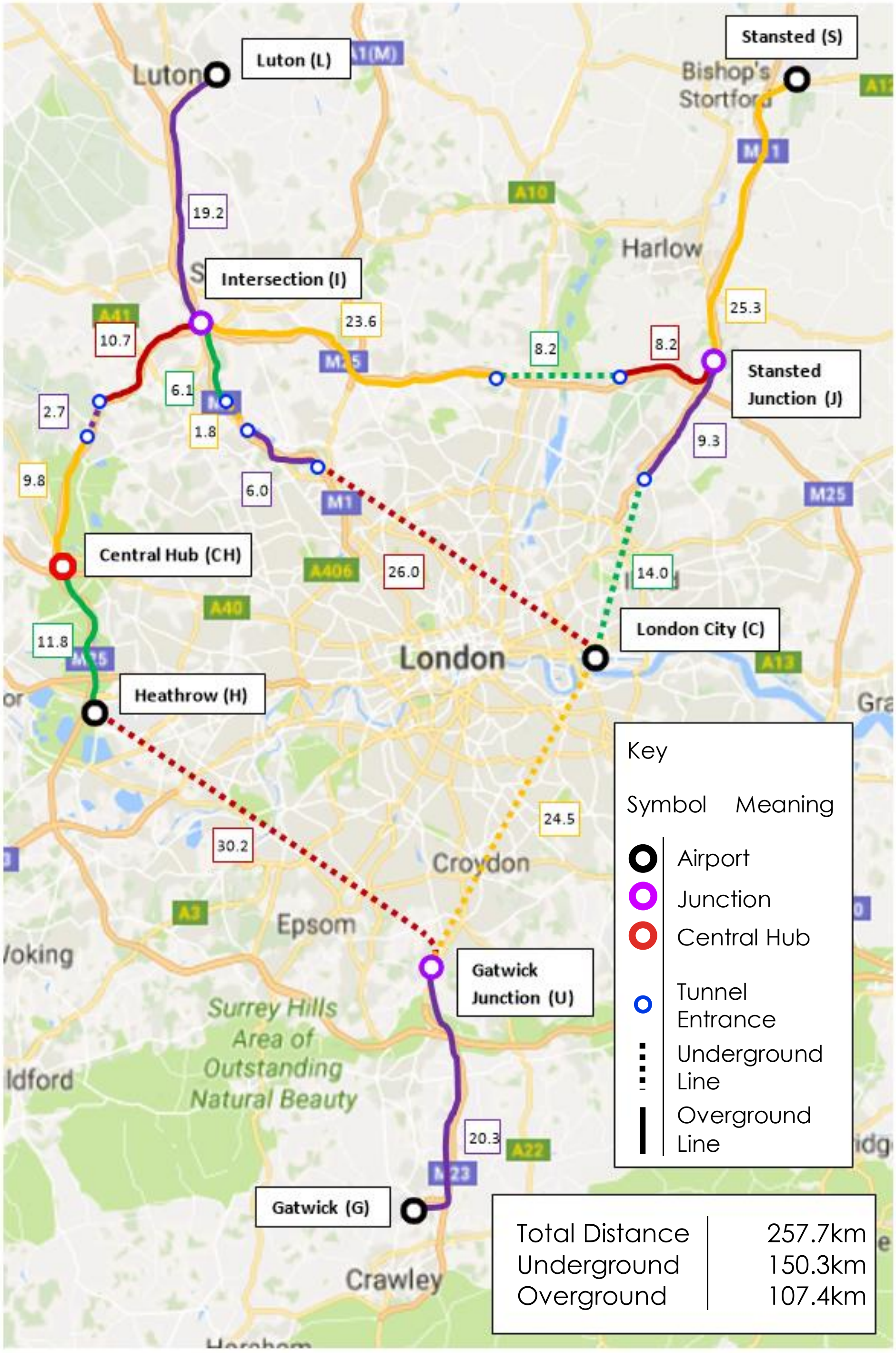


Line	Pros	Cons
Section A		
Green	Short and direct	Largely built up areas on either side, bad current connection to black
Red	Large open space for large part, smooth turn, connects smoothly to blue	The railway passes underground for approximately 1km bad current connection to Black
Black	Very direct, larger margin of space on either side of railway	Only connects up to 'Gatwick Linkup' Red Line A, can't connect to Heathrow well, bad current connection to Red/Green
Blue	Connects to red smoothly, has a large junction at the end	Bad current connection to yellow, smaller line with a station fully covering it at one point
Yellow	Connects to white smoothly, can easily lead off to Heathrow	Bad current connection to blue
	Connects to 'Gatwick Linkup' smoothly, can lead off to Heathrow	Not direct to London City
Pink	No turning, direct link to end of Gatwick Linkup	All underground, surfaces in Croydon
Turquoise	No turning, direct link to open area of Gatwick Linkup	All underground (more length underground than Pink)
Section B		
Green	Follows railway, leads to open area	Largely built up areas on either side for parts
Red	Connects railway options well	Poor angles of connection to other lines
Blue	Smooth curve, existing line, fairly direct	Not a large area of railway to work with
Yellow	Not many things in the way	Small turning radius, long way round to Heathrow
	Simple, short connection line	All underground
Pink	No turning, direct link to end of Gatwick Linkup	All underground
Turquoise	Connects to blue line nicely	All underground, requires connections between above ground and below ground lines in London
Orange	Direct link to London City	All underground, has large conflict with the Thames
Purple	Connects up the central lines well	Isolates the Gatwick Linkup

FINAL ROUTE

MAP

Having decided on all possible routes to be taken, the best for each section were selected, and distances were recorded for each part. A pickup point was added at the M40 and M25 intersection, called the Central Hub. More details and features of these can be found later. The final route map is shown below, and can be explored interactively at [1].



ROUTE SEGMENTS

The total route is split up into 10 different segments, each of which connects 2 of the 9 points (The 5 Airports, 3 junctions and the Central Hub). They are each named in the form A-B, where A and B are the two points they connect, in alphabetical order. The following is a brief summary of each segment, detailing length, number of tunnel entrances, and (if over-ground) any bridges and amount of Greenfield use.

Bridges are classified by number of road lanes crossed, counting the empty space in the middle of motorways and lay-bys as one or more lanes, depending on size.

The total area of Greenfield that is covered by the track is calculated by the following process:

- Find the length of overground track for each segment
- Subtract 3.7m for every road lane (or equivalently-sized obstacle) crossed.
- Multiply by 5m, the width of the footprint of each track (with one train going each way).

Track Label	Length $/km$		Tunnel Entrances	Greenfield Use $/ha$	Number of Bridges by Lanes Crossed				
	Overground	Underground			Total	1-5	6-10	11-15	16-20
CH-H	11.8	0.0	0	5.68	20	13	2	5	0
CH-I	20.5	2.7	2	11.33	27	26	2	4	0
L-I	19.2	0.0	0	9.35	32	27	3	1	1
I-J	31.8	8.2	2	19.55	52	40	6	6	0
J-S	25.3	0.0	0	12.40	24	16	4	4	0
H-U	0.0	30.2	0	0.0	0	0	0	0	0
G-U	20.3	0.0	1	9.90	26	20	2	2	2
C-I	12.1	27.8	3	5.85	17	12	1	2	2
C-U	0.0	24.5	0	0.0	0	0	0	0	0
C-J	9.2	14.0	1	4.52	12	8	2	1	1
Total	**150.2**	**107.4**	**9**	**78.58**	**210**	**162**	**22**	**25**	**6**

FEATURES AND ADVANTAGES

THE CENTRAL HUB

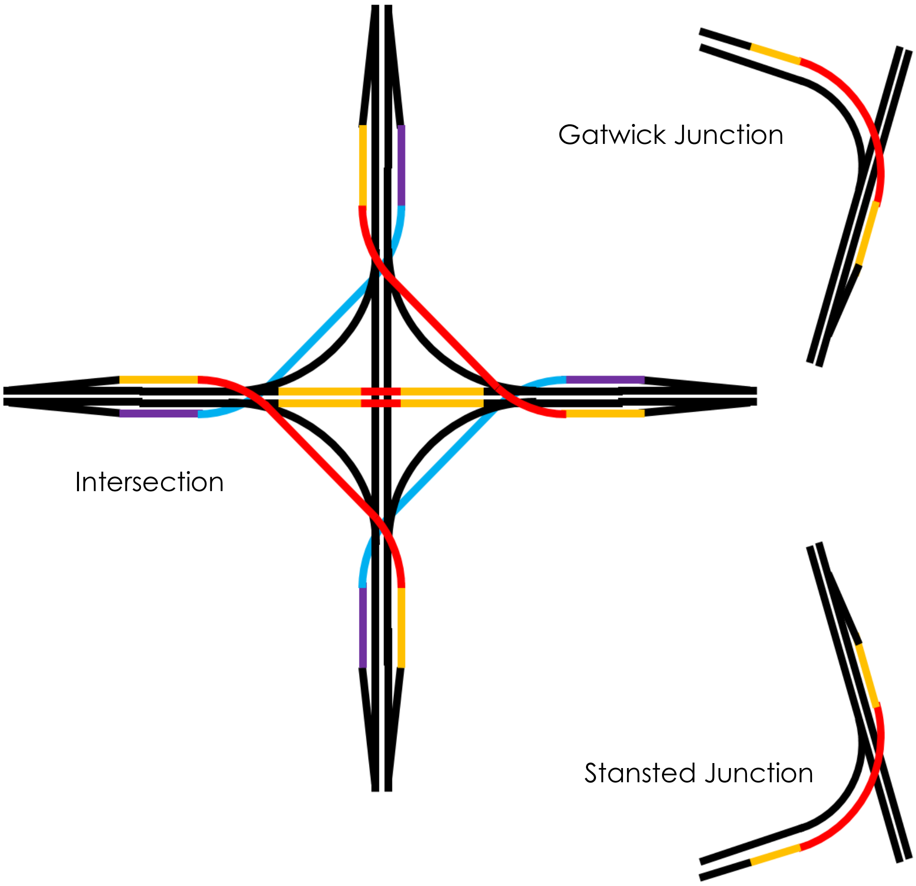
The Central Hub was added into the design at an early stage, one of its main functions being to alleviate congestion passing through Heathrow. The hub has a large car-park, allowing people to enter the network without needing to travel into Central London. This is more suitable than increasing parking availabilities within the outer airports, as that would only increase congestion through those areas, whereas a separate hub altogether can help spread out the traffic of passengers. Furthermore, it allows for easy drop-off and pick-up, being connected to two major motorways into and around London, the M40 and the M25, giving people ease of access to the system without having to be taken into London. This hub also acts as a storage facility for back-up and maintenance vehicles for the transport system, allowing a central point from which all help can be sent (more details about the role of the Central Hub discussed later).

ALTITUDE CHANGES

An analysis of distance required to change altitude for tunnels and bridges has been done in {1}.

THE JUNCTIONS

The Gatwick Junction, the Stansted Junction and the Intersection, are all places where the line splits, allowing trains from many different routes to use the same tracks. The Stansted Junction will be at ground level, once all tracks have returned back to the correct height from their bridges (365m from where they start going down {1}), in the open fields nearby, and the Gatwick Junction will be underground. The outer tracks in each of these junctions will loop over the continuing track at the height they would normally pass over roads.



Key of Track Heights

Blue	10m below normal height
Purple	Moving to below normal height
Red	10m above normal height
Orange	Moving to above normal height
Black	Normal height

With this intersection junction, the track travelling North/South in the centre will remain at the same height, and the one travelling East/West will rise to pass over the other line. The inner, tighter corners will be taken normally, whereas when tracks must be crossed, two of the tracks pass over the main tracks, and two pass under such that no two tracks of the same elevation are at any one point. The centre is 500m away from where the track splits for the smaller corners, which allows time for the track to split and then reach the correct elevation , and also allows the corners to be wide enough for the acceleration to not be too high. In order to make any tight corners in these junctions, the transport will slow down accordingly to maintain high passenger comfort levels.

THE LOOP

The fact that the whole system is a lopsided figure of 8 allows for potential backup services to run in the unlikely case of breakdowns. Although diverted journeys would take longer than 20 minutes, and increased route congestion would delay other lines slightly, the ability to maintain all possible connections in the case of a breakdown helps alleviate discontent from passengers, and avoids significant short-term financial loss.

GEOPHYSICAL FACTORS

INTRUSION INTO BUILT-UP AREAS

The chosen route allows for the whole transportation network to run either alongside roads - only going through fields - or to go underground. Throughout the entirety of the design, only a few small villages are passed, mostly on the I-J line. In addition, our system allows for highly built-up areas of London to be completely avoided.

GREENFIELD USE

We have chosen to have a route through mostly Greenfield land with our project. The advantages of this are that it is cheaper and faster to build on than already developed land, and it allows the most efficient and direct route to be taken because there are no existing constraints. Furthermore, it is a healthier environment, wherein the soil contains much less pollution and contamination than that found in urban areas, thus we release less pollution into the air through digging it up. Much to the same effect, Greenfield land often already shadows existing road networks.

The overall Greenfield use of the whole project is approximately 78.58 hectares, or 0.786 km^2, which is relatively small compared to the extensive road networks already present throughout London.

APPEARANCE

The trains have a sleek appearance and pass by quickly and so visual pollution is minimised. The idea of building a roofing structure over the entirety of the train tracks was discussed but rejected as such a permanent structure would make the system look far less appealing and add unnecessary costs.

NOISE

The trains are very quiet, as the lack of direct contact between train and tracks makes scarcely any noise. This means our system causes minimal noise pollution.

POLLUTION

There are no gas emissions directly from the train, so direct air pollution is absent. However, the total power demand is 2.37 petajoules each year, which is approximately the equivalent of 79,000 tonnes of coal being burned every year as just 1 tonne of coal releases about 30 GJ when completely combusted (depending on the exact composition and quality). This causes both significant atmospheric pollution and a burden on the National Grid.

ACCESS AND FUTURE EXPANSION

ACCESS

The system, before any future expansion occurs, will have 6 access points – the 5 airports and the Central Hub. Some proportion of those travelling in from the North can be taken into Luton or Stansted, but the majority be likely to access the system via the Central Hub, as it has better parking facilities, and better surrounding road infrastructure. Western travellers will access via the Central Hub, Southern travellers will access via Gatwick, and Eastern travellers will either go to Stansted, London City or Gatwick, depending on which is easiest to access.

Access via train is possible by taking the London Tube Network and DLR from London Paddington Train Station to London City Airport. The system can then be used as normal from there. Trains can also easily be taken to Heathrow from London Paddington.

COMMUTER USE

Before any future expansions, the system usage is predicted to be an approximately half-half split between airport users and commuters. More commuters can be accommodated for with future expansions or increased carriage numbers on less congested routes, should demand rise.

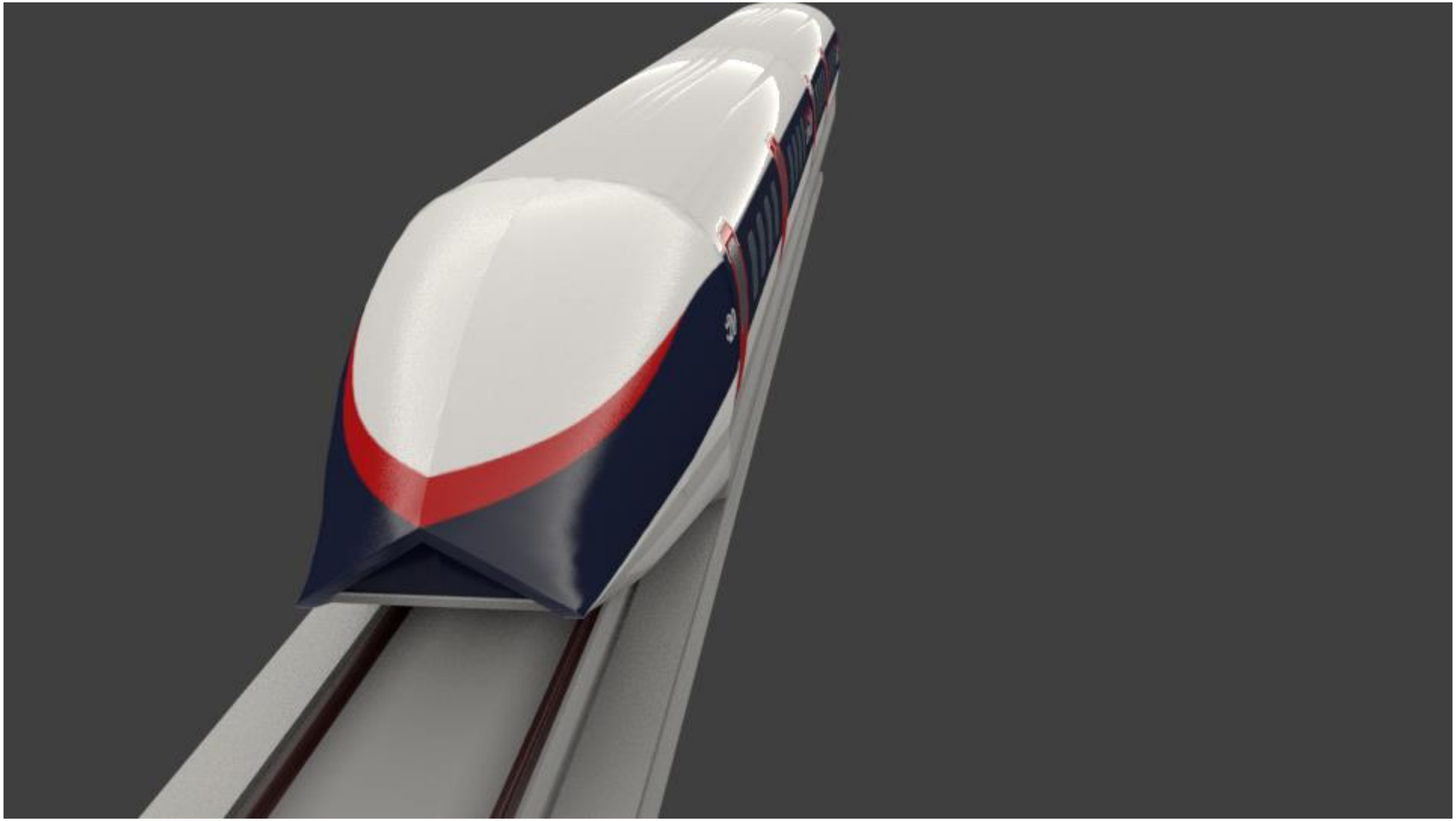
FUTURE EXPANSION

Our system has various aspects that can be expanded, depending on what is most necessary after the whole system is constructed:

- Multiple tracks can be constructed for each line, allowing for potentially double the flow of passengers between airports.
- On less congested routes, the number of carriages per train is purposefully reduced to increase efficiency. If demand on these routes increases, it is trivial to increase carriage numbers.
- Other train services can be improved and incorporated into the network. Although incorporation of the Great Western Railway into the initial design was decided against, it is certainly a possibility to integrate it into our network later, due to the convenient location of the Central Hub.
- Due to the burden that would be present on the National Grid because of the energy demand of the system, the National Grid itself could potentially be developed to account for this.

- Likewise, further and improved connections can be made with London Paddington Train Station. This would allow for a much larger use of the system by commuters travelling to London by train, as well as letting airport passengers access the system even more easily, by removing the necessity of using cars or buses.

Within a few years of the completion of the whole design, these concepts will be reviewed, and any suitable expansions will be implemented to improve the design of our system.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ENGINEERING

IDEA REFINEMENT

Due to the volume of traffic between the airports and the speeds required to meet the constraints of the task, there were only 3 possible methods of transport available to us. These are conventional high speed rail, magnetic levitation, and a hyperloop system.

The hyperloop was an idea proposed by Elon Musk using a network of tubes containing passenger cars. The tubes are partially evacuated and maintain a pressure of a thousandth of an atmosphere to reduce air resistance. Furthermore, each car has a front mounted compressor fan to prevent pressure build up in front and channel the air backwards.

Part of this air flow is used within the car for ventilation and also to cushion a series of air bearings which are used to reduce friction between the car and tube walls. The combination of vacuum tubes and low friction means there is very little resistive force acting on the car (320N drag when travelling at 700mph). The car can therefore be accelerated up to high subsonic speeds, with sections of linear motor spaced periodically along the tube, and coast for the rest of the journey [2].

The idea of a hyperloop was abandoned early on because of the cost in producing vacuum tubes of the length required, safety issues in case of emergency, and the fact that there were no current implementations of the technology available from which we could get an idea as to the performance and costs of the system.

The hyperloop option did, however, present some significant advantages, such as drastically reduced energy usage as well as greater top speeds due to the elimination of most friction and drag. Furthermore, it would have relatively low maintenance costs because of the lack of contact between the cars and tubes.

High speed rail (HSR) was also considered for the system. HSR is fundamentally much like conventionally rail systems, however, the rolling stock and track are specially designed to cope with the higher speed and challenges it poses. It is currently the primary choice for the UK government with both Crossrail and HS2 using it. The current fastest commercial HSR implementation is the TGV in France which has a top speed of 320km/h [3].

The primary advantage of a HSR solution was the relatively low implementation costs including the ability to reuse pre-existing rolling stock and track. Another reason for the use of HSR would be that existing routes can be used and, while this is less relevant to our scenario, it is a consideration that must be made, because it can make it possible to increase the capacity of the transport system at short notice, should the traffic volume be too high. For example, there are National Rail lines connecting all airports except London City and the DLR and Piccadilly lines on the Underground also connect Heathrow and City [4].

HSR also has disadvantages. Most importantly is that, due to mechanical wear, operating and maintenance costs are very high: around £160,000 per route mile per year for track and infrastructure [5]. Additionally, rolling stock maintenance would amount to over £1.49 billion on our system {2}. There are also environmental concerns, such as the noise pollution and high energy usage that are common to HSR. Finally, there is a reliability concern. HSR systems are easily affected by adverse weather, such as ice, snow or heavy rain causing flooding. It would be desirable to avoid these issues.

The final option considered was a maglev-based system. Unlike HSR, maglevs do not make physical contact with the track; they rely on magnets to keep them levitating. Consequently, there is none of the friction and mechanical wear characteristic of HSR. This reduces operation costs significantly.

Transrapid, a German high-speed maglev network, will be used for the majority of comparisons, in which maintenance costs are much lower than those of HSR [6]. Another example of where mechanical wear should be considered is in braking – brake discs in high speed rail are frequently worn down, whereas in a maglev system, cars would be slowed by the magnetic field.

The lack of friction also means less noise pollution with a maglev system. Transrapid has demonstrated that maglevs offer a noise reduction of just over 10dB (A-weighted) and, when cruising at 200km/h, they are barely audible [7]. This also improves passenger comfort because there is no vibration and consequently a smoother ride.

Maglevs can also reach significantly higher speeds than HSR, with Transrapid reaching 500km/h [8] and the current speed record of 603km/h going to the Japanese L0 Series [9].

From a reliability perspective, maglevs are again a good choice. They can cope better with ice and snow than HSR because they do not need direct contact with the track. In Japan, they achieve 99.98% success with being on time and, in Germany, they have been in use since 1987 with no serious failures/accidents [10]. Finally, maglevs can climb steeper gradients than HSR (10% as opposed to 4%, [6]). This allows junctions in the network to be reduced in size, because the car can begin elevation closer to the actual junction.

Maglev systems do, however, have their disadvantages. Firstly, and most importantly, there is a greater upfront cost as a result of more complicated track designs. This can make large routes harder to implement and is why many maglev systems form smaller linkups. However, it has been demonstrated by Transrapid that a large network is not impossible. This cost is also partly due to having to implement an entire new system without the ability to use current rolling stock and track.

Lastly, with some implementations, there is a safety concern with power cuts. In summary, if the levitation system loses power, the car can collide with the track. This issue will be covered in more detail in the next section.

All things considered, it was our decision to go with a maglev-based system because of the improved speeds and reliability. Furthermore, the reduced operation and maintenance costs will make the system significantly more profitable in the long run.

The following section gives a technical description of our system and explains how we arrived at it.

SYSTEM DESIGN

There are 3 key components to a maglev system:

- Levitation system
- Propulsion system
- Stability system

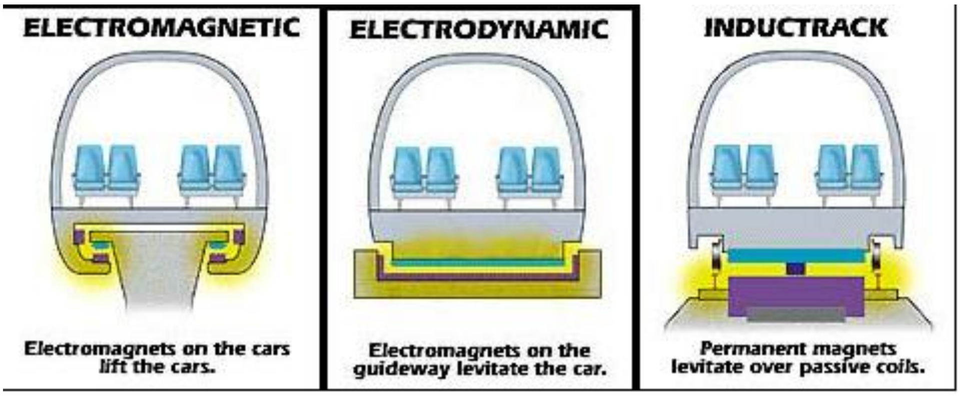
They will be discussed in order below.

LEVITATION SYSTEM

For the levitation system, there were 3 primary options that we could go with. These are:

- Electromagnetic suspension (EMS)
- Electrodynamic suspension (EDS)
- Inductrack

Of these, only EMS and EDS have previously been implemented whereas inductrack remains a concept.



Comparison of suspension systems. Source: http://students.iitk.ac.in/projects/eclub_maglev

EMS relies on electromagnetic coils attached to the train underneath the track for levitation. These attract to the track lifting the train. This system is the simplest to implement and is currently used in the low-cost Transrapid network [8]. It is also a very safe system because the train 'wraps around' the track. This prevents derailing should something go wrong. Finally, it uses relatively weak magnetic fields, which makes it safe for passengers with pacemakers, and also means magnetic shielding is not required.

The main disadvantages choosing EMS are that it draws a significant amount of power to power the levitation magnets, increasing the energy costs for the entire system, and through this causing a larger environmental impact. Implementations of EMS also require a complicated control system to keep the train positioned correctly.

As stated by Earnshaw's theorem, no system of magnets can be in equilibrium unless mechanically constrained in at least one axis. This means the current through the levitation and stabilisation electromagnets in an EMS system has to be constantly adjusted according to the distance between the train and track which requires the addition of sensors (probably ultrasonic) across the length of the train, electronics capable of smoothly controlling the current flow (since coarse control such as turning on and off the coils would be uncomfortable for passengers) as well as a computer to monitor the sensor data and calculate the required currents accordingly.

This also introduces an extra point of failure into each train and any fault in it could cause the entire train to crash into the track. On this note, another disadvantage to an EMS-based system is that any power failure would immediately disable the levitation system, allowing the train to crash. Transrapid combats this issue by installing batteries inside each train carriage to continue powering the levitation system in case power loss occurs [8].

EDS relies on superconducting electromagnets within the train to repel levitation coils located within the track. This is the design employed by several high-speed Japanese maglevs, including the experimental Chūo-Shinkansen maglev currently in testing and the L0 Series maglevs. The coils in the train are powered and induce a current in levitation coils in the track. To increase efficiency, the coils in the train are cryogenically cooled by liquid helium. Since the field in the levitation coils is created from the induction, Earnshaw's theorem does not apply and EDS is inherently stable [11].

The primary advantages of EDS are the speed that can be achieved and the energy efficiency of the system. As demonstrated by the Japanese maglevs, EDS can be used for speeds up to 600km/h [12]. EDS is also more efficient than EMS because the powered electromagnets are superconducting. This is also a safety advantage for EDS because, in the event of a power failure, current will continue to flow inside the electromagnets and the train will continue to levitate. Finally, EDS generally results in less noise pollution, as the side walls of the track have the effect of reflecting most sound back towards the train.

The main problem with an EDS system is the large cost associated with it, particularly for the cryogenic system, which uses liquid helium. This is a limited resource and prices will only rise with time. The cost penalty is demonstrated by the implementation costs of various maglev systems given below:

ITEM	COST
TRANSRAPID SYSTEM (EMS)	£40m per km
SHANGHAI TRANSRAPID SYSTEM (EMS)	£87m per km
CHŪO SHINKANSEN (EDS)	£240m per km
TRANSRAPID CARRIAGE (EMS)	£10.3m
L0 SERIES CARRIAGE (EDS)	£89m

Costs adjusted for inflation and converted to GBP [9], [13-15]

Other negatives of EDS include that trains do not levitate until reaching 100km/h, which results in mechanical wear of wheels and brakes, and the fact that strong magnetic fields created by the superconducting magnets means a lot of money has to be invested into magnetic shielding.

"Inductrack" is the final method for achieving electromagnetic levitation. This is similar to EDS, although it is a passive technology. Permanent magnets mounted on the bottom of the train induce a potential difference in passive, short-circuited coils beneath the train, as described by Faraday's law of induction. When moving fast enough, peaks in the magnetic field from the permanent magnets on the carriage line up with those from the coils, creating a strong repulsive magnetic field and levitating the train.

This system was initially considered unfeasible, due to the lack of sufficiently strong permanent magnets. However, it has been demonstrated that with neodymium magnets arranged in a Halbach array, the system is possible. A Halbach array is a special arrangement of permanent magnets that maximises the magnetic flux on one side while reducing it on the other side. Building a Halbach array from neodymium magnets creates a strong enough magnetic field to achieve levitation at relatively low speeds. While inductrack has not yet been used in a commercial maglev system, a full-scale model has been constructed, which achieved separation distances from the track of 25mm when moving at only a few km/h [16].

The main advantage of inductrack is that it is essentially an EDS system and should be capable of similar speeds, however, without the need for the cryogenic system, initial and running costs are both significantly lower. It will also remain levitating until it slows in the event of a power failure, increasing safety.

Another important advantage is its efficiency. At speed, the power loss from magnetic drag in the levitation system is only 0.5MW which is significantly less than aerodynamic drag at these speeds. [17] This is not the case for active EDS or EMS and this makes inductrack the most efficient choice for our system.

Disadvantages and difficulties with an inductrack system include the fact that there are currently no commercial implementations that we can study to make decisions for our own system. There is also a greater upfront cost than for an EMS system, and the greater track complexity will increase construction time.

We have opted to implement an inductrack system for the reason that it is more efficient and has lower operation costs as a result. This will make our system more profitable in the long run.

PROPULSION SYSTEM

It was suggested by the inductrack document that jet engines could be used to propel high-speed inductrack maglevs. The disadvantages associated with this propulsion method caused us to discount it.

- Safety concerns storing jet fuel on board
- Noise pollution & Greenhouse gas emissions
- Uses up oxygen in tunnels

Added carriage mass form fuel and engine

The other option was to use a linear motor as in most current maglevs. There are two types of linear motor available:

- Short-stator linear induction motor (LIM)
- Long-stator linear synchronous motor (LSM)

A LIM consists of a short stator (the section containing the powered coils) and a reaction strip often made from aluminium. The powered electromagnetic coils on the train induce a current in the reaction strip creating a magnetic field to repel and propel the train. This is the system in operation on most low-speed maglevs including the Linimo maglevs in Nagoya, Japan. [18]

A LIM would be the cheapest option to implement because of the short stator design. This significantly reduces track costs because coils are not needed along the entire length of the track. Furthermore, power delivery to the maglev is simplified by using conductive rails with a conductive brush like in a conventional dc motor to deliver power.

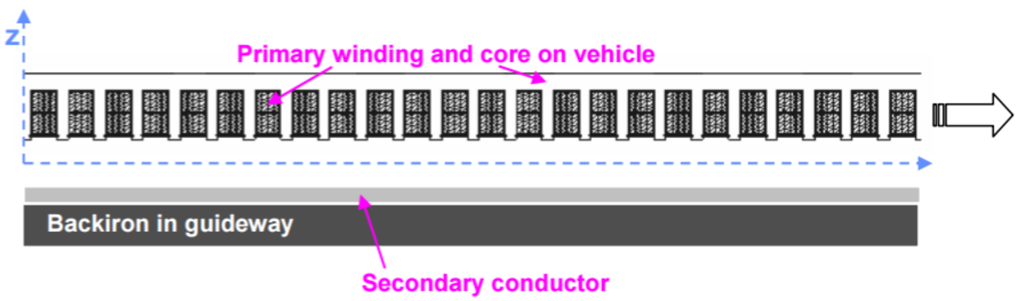
The disadvantage with using a LIM is that this power delivery system is not capable of being run at the high speeds our system requires. Brushed power delivery has been tested up to speeds of 130km/h in the Nagoya test track. When using wheels as brushes instead of sliding metal contacts, speeds of 200km/h have also been achieved [18], however, this is still not sufficient for our system. The reason behind this is that friction between the train and track wears down brushes quickly at high speeds, resulting in unreliable contact to deliver power. LIMs also tend to be less efficient than LSMs, which affects the energy usage and therefore operation costs of our system.

LSMs operate using a stator built into the track. This creates a field that can repel permanent magnets on the vehicle or, in the case of Transrapid, the levitation magnets [18]. This requires communication between the train and track to ensure the stator is powered correctly as the train passes over it. This must be done with a very high precision to ensure the movement of the train is smooth and the ride comfortable.

The fastest maglev networks all use LSMs for propulsion and the technology has been demonstrated to work up to 500km/h. This is because no contact is required between the track and train and all power conditioning equipment is located track-side, resulting in lighter vehicles and therefore allowing speeds greater than those possible with a LIM. Furthermore, efficiency of an LSM-based system is greater, especially when the block length (the length of a discrete powered "unit" of track) decreases. As block length increases, efficiency is less because impedance increases.

The disadvantage of usage of LSM is the associated cost. Track design is significantly more complicated because of the addition of the extra propulsion coils, which is expensive for large networks. Furthermore, power delivery is more expensive, because each train on the track requires an additional substation. This is because each substation has to deliver the 3-phase power to the track in time with the train it is controlling.

Despite the costs, we use a LSM for our network because a LIM would be unfeasible at the speeds required.

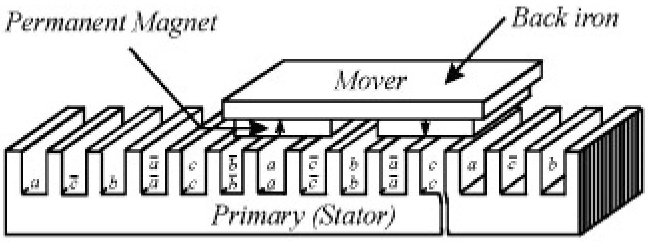


Cross-section of a Linear Induction Motor. Source:
https://ntl.bts.gov/lib/24000/24600/24692/FTA-DC-26-7002-2004-01.pdf

Linear Synchronous Motor. Source:
https://sites.google.com/site/km4774/maglevtraintechnologies

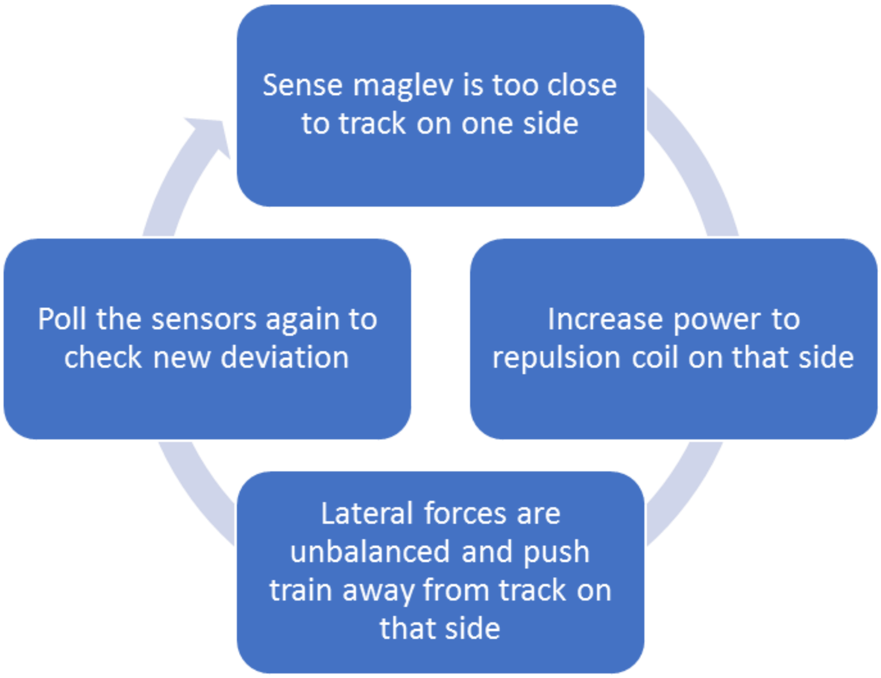
STABILISATION SYSTEM

The stabilisation system of a maglev is responsible for keeping the train positioned correctly on the track. It is also necessary to turn at speed. The stabilisation system provides the lateral



forces to guide the train along the centre of the track and therefore, as the track turns, the train will too.

In Transrapid, it consists two coils in the sides of the track that repel the train to keep it central. This is particularly important in EMS-based maglevs, because they are inherently unstable, so it is necessary to precisely monitor the distances between the train and track. This information can then be fed to the stabilisation system to adjust the power of the stabilisation coils and counter any deviation of the train from the centre of the track. In all maglevs, the stabilisation system forms a simple feedback loop:



Since we are using inductrack for levitation, our system is stable. However, a stabilisation system is still necessary to provide sufficient turning forces and for safety purposes.

ROLLING STOCK

The vehicles that run on our system are automated trains composed of carriages that each carry the necessary levitation system. The motive power is provided by the track, in the form of a magnetic field it induces in front of and behind the train.

CARRIAGE TYPES

There are two types of carriage in a train; middle-section passenger carriages and end-section equipment carriages. Since the system is automated and controlled by a computer system, the end-section carriages do not contain a driver's cabin. Instead, the end sections are used for carrying batteries and A/C units for the passenger carriages, as well as various instruments such as communication and signalling equipment for train control. These end carriages are aerodynamically streamlined to reduce drag and contain air intakes to provide an air supply for the entire train.

Diagram	Passenger Carriages	Capacity
	1	57
	2	114
	3	171

A train can be adjusted to have different numbers of middle section passenger carriages, which increases the flexibility of the vehicles and opens up possibilities for future expansion.

Passenger carriages have the capacity of up to 57 passengers, and a train can have a minimum of one passenger carriage (with two end section carriages connected on either side). We will use up to three passenger carriages at once, as more carriages could disrupt the stability of the train when travelling at high speed, and three carriages are sufficient for the maximum peak capacity. The interface between the carriages allows the connection of power cables, signal cables, pressure relief ducts and ventilation ducts.

Each passenger carriage will contain enough space for baggage from 57 people. This makes carriage production and design simpler, as they are uniform, and makes it easier to change the carriage number for a train during the day to cope with demand.

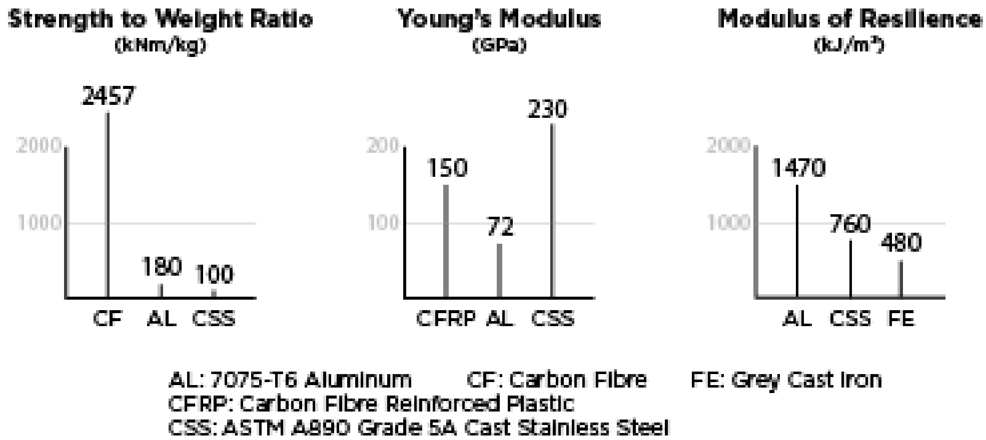
CARRIAGE FRAME

Because of the short journey times and the constant acceleration/deceleration being experienced, we decided it was safer not to provide services such as toilets and catering. This also helps cut down on carriage mass and expenses. Instead, passengers with wear seat belts and remain seated for the duration of the journey.

Carriages will be designed in a semi-monocoque [19] structure, consisting of a skin, stringers and bulkheads. This design has been employed in aircraft for years and is also used in the Falcon 9 rocket by SpaceX [20] as well as the Bloodhound SSC [21]. The reasoning behind looking to the aerospace industry for the design of our carriages is that aircraft experience similar conditions to those our maglevs will. At speed, aircraft can experience high pressure build-up especially during take-off and landing.

Additionally, the carriages must be airtight to maintain cabin pressure. This is also true of our network, particularly for underground sections, where the cabin pressure will have to be kept constant despite pressure waves, in order to ensure passenger comfort. This form of construction also reduces the weight of the vehicle drastically, therefore improving the efficiency of the train.

Materials commonly used in the aerospace industry for this design include steel, titanium, aluminium and magnesium alloys and carbon fibre reinforced polymers (CFRP) [22].



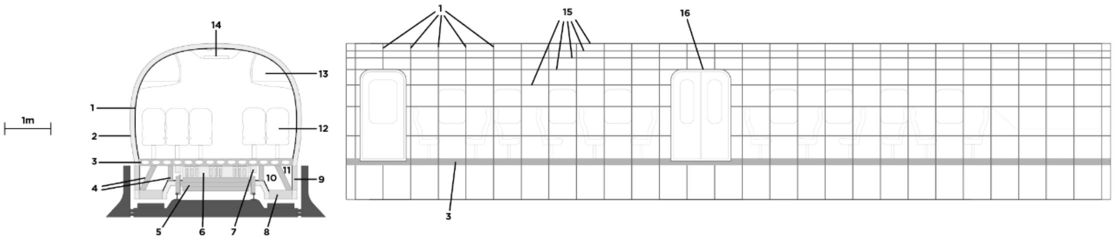
The bulkheads are made from CFRP, similarly to the A350 XWB aircraft [23]. CFRP has a very high strength to weight ratio, and can be easily weaved into the shapes required for the carriage, eliminating the need of multiple components and thus points of failure at joins. It is a composite material consisting of a matrix such as epoxy resin and the carbon fibres. CFRP exhibits directional strength properties because of the orientation of carbon fibres within and, while it may have a high Young's Modulus when the force is applied in the direction of the fibres, it can be significantly lower if the load if the fibres are oriented transverse to the load. It is common for carbon fibres to be woven into a fabric to combat this, allowing multiple layers (often two at 45 degree offsets) to be used. [24] Consequently, in our system, to ensure rigidity and strength in all directions, parts are produced using multiple carbon fibre layers of different orientations.

Toughness of CFRP is also a consideration. Toughness is a measure of how much energy can be used to deform a material without fracturing. In CFRP, this is highly dependent on the matrix used. Toughness is important to us because should anything hit the train, it is important that it absorbs the impact. We expect that, in the coming years as CFRP is more widely adopted both inside and outside the aerospace industry, new matrixes will be developed and used with increased toughness. Furthermore, it is also likely that prices of CFRP will also decline so we estimate a cost of ~£12 per kg [25].

The stringers [26] along the length of the train connecting the frames and the skin of the carriage are aluminium alloys. These are well-studied materials that are highly reliable. They have a relatively high strength to weight ratio whilst also being malleable [27], which allows them to be curved into the shape of the frame.

Generally, 7075 aluminium is the alloy used in aircraft [28]. It is a very high-strength material often used in structural components and it is also very stress-corrosion resistant. Furthermore, depending on the temperature used to temper it, it can be easily machined making it easy to form into the shapes required. On the skin of the maglev, the forces experienced by the aluminium will be shear forces between the bulkheads. We performed CFD analysis to determine the pressure experienced which gave a maximum of 180kPa at the head and 25kPa along the carriages. (See Appendix for details, {3}) This can be used to calculate the maximum spacing between the CFRP bulkheads which shall be 0.15m for the head of the train and 0.6m for the passenger carriages {4}.

An exterior that is made of aluminium acts as a safety feature – aluminium has a high electrical conductivity, so if the vehicle were to be struck by lightning, the exterior would dissipate and ground the current through the wheels [29], without endangering the carbon-fibre interior and the passengers inside.



1:CFRP Frame 2:Aluminium Skin 3:Floor Truss 4:Support Truss 5:Auxiliary Wheel Set 6:Shock Absorbers 7:Emergency Breaking Coils 8:Halbach Array 9:Stabilising Coil 10:Space for Pressure Relief Duct 11:Space for A/C Duct 12:Seats 13:Overhead Luggage Compartment 14: Lights and Wireless Routers 15:Longitudinal Stringers 16:CFRP Door Frame

FLOOR

Directly above the levitation system and the auxiliary wheel sets, there are two layers of supporting truss (a structural framework that is partially hollow, reducing the weight of the structure) made from CBRP. These being made from strong material is necessary since they support the weight of the carriage. They enclose the mechanics of the train and also form the base on which the flooring is installed and the passenger seats are fitted.

Underneath the layer of truss are the cables for power supply and signalling, alongside two air ducts. The use of these is discussed in the Carriage Mechanics section.

CARRIAGE INTERIOR

The upper layer of the truss also acts as the floor of the passenger cabin, upon which the seats are fitted and the corridor is paved. Luggage compartments are situated at the two ends of the carriage. They have a sliding roof that can open up for loading and unloading of luggage.

The interior of the cabin is lined with high-temperature thermoplastics, which are both water and heat resistant [30], making the interior of the cabin easy to clean and damage resistant, as well as giving it a modern and appealing visual, similar to an aircraft. Taking inspiration from the aerospace industry [31], we use polyphenylsulphone (PPSU) because of its strength and chemical resistance. We have budgeted 1500kg for thermoplastics which equates to 1.15 m^3 (using a density of 1300 kg/m^3 [31]) for walls and other parts such as luggage racks.

In between the interior thermoplastic shell and the carbon fibre frame of the vehicle, a layer of insulating foam is present [32]. This serves two purposes:

- It acts as a thermal insulator, which increases efficiency of the A/C system
- It also acts as a firewall, preventing the spread of fire in an emergency.

The flooring is a non-slip material, as in other forms of public transport, in order to improve passenger safety.

Windows are installed in the skin between bulkheads. The window panels are large sheets of reinforced polycarbonate (PC), which are connected to the CFRP frames by pressure-tight seals. A rubber seal attaches and seals the window panel to the frame, which in turn is riveted to the frame of the cabin. A dust cover keeps the window panels clean, and acts as a double glaze for thermal insulation. PC is a material that is very strong and simultaneously highly inert [33]. This allows it to resist weathering for long periods.

COST BREAKDOWN

A breakdown of individual materials is given the following tables for both carriage internals and structural components.

ITEM	INDIVIDUAL MASS /KG	INDIVIDUAL COST /£	COUNT	TOTAL MASS /KG	TOTAL COST /£1000
PASSENGERS	85	-	57	4845	-
SEATS	9	2100	57	513	119.70
LUGGAGE	23	-	57	1311	-
SAFETY EQUIPMENT	-	-	-	10	0.15
FLOORING	2.5	20	42.5	106	0.85
THERMOPLASTICS	-	-	-	1500	50.00
INSULATION	-	-	-	10	2.00
TOTAL	-	-	-	8295	172.70

The average adult mass for North America was used because it is currently the highest and it was rounded upwards to allow for extreme cases [34]

The Recaro SL3510 was used for mass estimates. This is a slim and lightweight aeroplane seat which makes it comfortable for high density seating because it still leaves plenty of leg room and it has won innovation awards because of this. [35] Price estimates are according to an FAA report [36]

Floor pricing and mass is given per m^2 [37]

Values are estimated for thermoplastics and insulation

Breakdown of carriage structure:

ITEM	INDIVIDUAL MASS /KG	INDIVIDUAL COST /£1000	COUNT	TOTAL MASS /KG	TOTAL COST /£1000
CFRP BULKHEADS	23	100	27	621	2700
7075 SKIN	3.4	4	200	680	800
7075 STRINGERS	-	-	-	230	670
PC WINDOWS	1.5	5	50	75	250
CFRP TRUSS	50	100	33.2	1660	3320
TOTAL	-	-	-	3266	7740

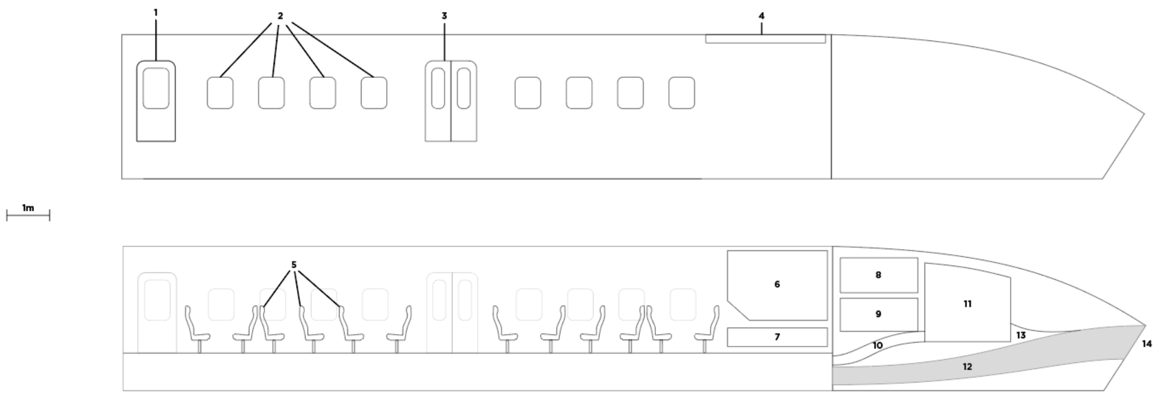
Bulkheads were approximated as a 10x10cm H-beam of 5mm thickness running around the walls and roof of the carriage. Density 1800kg/m^3 [38]

Individual values per m^2

Windows assumed to be 8x400x400mm panels

Truss values given per m

This gives a total cost of **£7.74 million per carriage**, and a mass of **11561 kg** when filled.



1:Sliding Door 2:Windows 3:Double Sliding Door (for wheelchair access) 4:Sliding Roof 5:Chairs
6:Space for Checked-in Luggage 7:Emergency Battery 8:Main Battery 9:Control Unit 10:A/C Duct
11:A/C Unit 12:Pressure Relief Duct 13:Air Intake for A/C Unit 14: Air Intake

CARRIAGE MECHANICS

The mechanics of the passenger carriage can be broken down into these sections:

- Halbach array
- Auxiliary wheels
- Environmental control e.g. AC/Heating
- Communication
- Power delivery
- Stabilisation system
- Emergency Brakes

The Halbach array is necessary for both levitation and propulsion of the maglev. It is constructed from neodymium N52 grade magnets. These are among the strongest rare earth permanent magnets which is necessary for sufficiently strong levitation and propulsion of the trains. One problem with using neodymium magnets is that they are susceptible to corrosion. [37] To solve this issue, the Halbach arrays are mounted internally behind the skin so they are not exposed to the elements. Furthermore, they are nickel plated to galvanise them.

There are two Halbach arrays per carriage which straddle the track. Each is 20cm thick by 80cm wide and extends for the length of the carriage. By positioning them in this way, it provides a wide base to improve stability when levitating. Since there is some repulsion force even when moving slower than the take-off speed, there is also be improved stability when the train is running on wheels. Each array is fastened securely to the CFRP truss because the entire weight of the train must be supported on them. For cost and mass estimates, we assume neodymium magnets to cost about £95/kg [39] and have a density of 7400kg/m^3 [40].

Auxiliary wheels are also necessary for when the train is not levitating. Like on conventional trains, these are mounted on bogies. A bogie is an assembly consisting generally of 4 wheels, a steel frame to distribute load between the wheels and suspension on which the train runs. Suspension will be air based for maximum comfort. The bogies have 30° of rotational travel to allow the train to turn corners when not levitating. The auxiliary wheels and bogies are constructed from steel, similar to that of conventional high-speed rail rolling stocks, this choice of material is made because of its stiffness and strength, which allows it to withstand the contact pressure from the train's weight without deforming. [41]

Component	**Power Draw /W**
Lighting	240
Heating	1200
AC	2000
Communication	20
Other Systems	100
Total	3560

Environmental control includes air conditioning, heating and lighting on the trains. Lighting uses LEDs for maximal efficiency. We have also decided to use cooler colour temperatures for lighting. This helps passengers to be more alert, [42] which ensures they are prepared for any connections they have to make. We assume LED bulbs to consume 8W each with approximately 30 per vehicle for a total of 240W. These are situated in the top of the passenger cabin, which has an indent running through the length of the train to carry the ceiling lights, telecommunication devices, speakers for the PA system and their respective cables. Heating is supplied per carriage and draws 1200W {5}. This is accomplished by two electric heaters at the front and rear of the carriage located in the centre just under the floor.

Finally, ventilation and air conditioning is required to cool the maglev and circulate new air. Air for ventilation will be taken in through a vent on the front of the train. This will be travelling at too high a speed to be usable inside the train and will therefore be slowed by a series of baffles within the air ducts. This is the same system employed in the Bloodhound SSC to slow supersonic air down for use in an internal combustion engine [43] and should therefore be more than sufficient for our needs.

Air ducts are made from an aluminium alloy. They carry air from the front of the train along its length and to the rear where it is ejected. There are separate ducts for cabin air and cooling air since cabin air should travel at a lower velocity than air for cooling. Like ventilation, air conditioning is also handled in the front of the maglev. This is because AC systems can be bulky it is more cost and weight effective to install a single larger unit rather than multiple units in each carriage.

Communication is an important aspect of our system because all control is autonomous and handled remotely at our central hub. Given the speeds of the trains, it is vital to have reliable network for communication. This is because, in the event of a failure, all of the trains must begin decelerating without hesitation to avoid collisions.

There is one exception to this which is when this would cause the train to stop on a banked corner. In this instance, train speeds will be carefully controlled to maintain a sufficient speed to travel around the corner with the perceived centrifugal force keeping the train in place on the track.

To allow this to happen, gaps between successive trains will be kept such that the one behind would be allowed 5 seconds of 'thinking time' before braking to come to stop without crashing. This allows for slight lapses in communication since communication losses are not impossible. If a communication gap is longer than 5 seconds, the central hub must assume that there is a serious fault in which case the entire system must be stopped for safety reasons. Furthermore, if any car loses connection to the central hub, it too must respond by coming to a halt. Because of this, efforts must be made to reduce the chances of communication loss.

To mitigate this problem, our maglevs contain two transceivers each (one in either end section of the train) to provide a backup should there be an issue with the primary. Given our reason for usage, we will reserve a frequency band in the 900MHz region. This band is currently reserved by Ofcom for mobile communications [44] usage for 2G networks. This should provide more than sufficient bandwidth for our system (40-50kbps, [45]).

The 900MHz frequency band has been chosen since 2G is already scheduled to be phased out by AT&T this year [46] and it is likely that, by the time our system is implemented, it will also be phased out in the UK. This will free up that band and it could also enable us to buy up some of the existing 2G infrastructure for less than it would cost to implement ourselves. As a contingency, however, these costs have still been considered.

For the transceivers on our maglevs, there is an 8W transmission power. This is 4 times greater than is traditionally used in mobile phones [47]. The transceivers we use also have lower noise, higher quality amplifiers which gives them improved sensitivity and selectivity over those found in mobile handsets. For example, cavity filters will be used to filter out all noise that is not the desired frequency. Given inefficiencies in the system, we anticipate a power draw of 20W for communication.

Power will be supplied to the trains inductively when in motion. This is achieved through coils mounted on the sides of the train, into which coils in the track will induce a potential difference. Given previous on-board power estimates as well as an additional allowance of 100W for control computers on board, we estimate a total power requirement of 3.6kW.

The levitation system is discussed in more detail and simulated in {6}, showing that the train will lift off the ground at a speed of 20 km/h, or 5.55 ms^{-1}. Battery requirements are discussed in {7}.

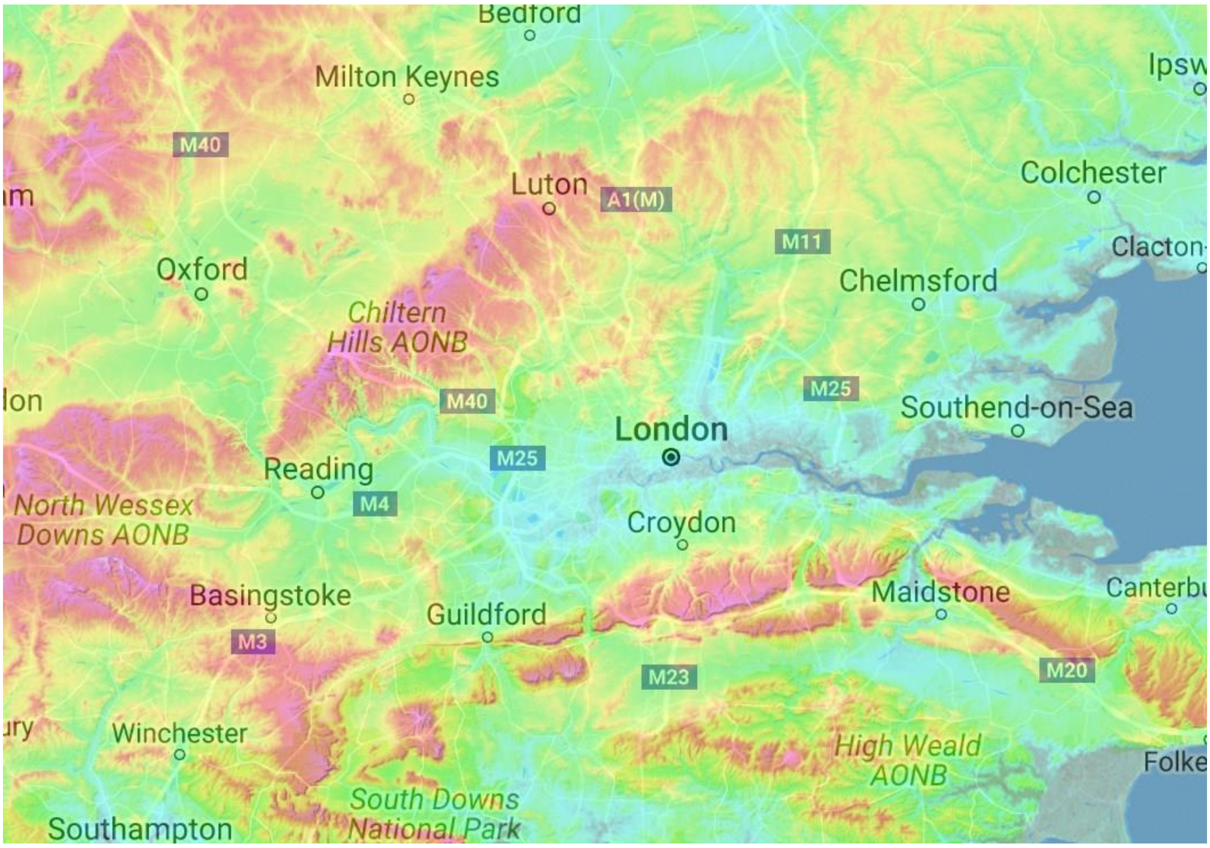
Item	**Individual Mass /kg**	**Individual Cost /£**	**Count**	**Total Mass /kg**	**Total Cost /£**
Halbach Array	7,400	703,000	2.7	20,000	1,900,000
Batteries	-	-	-	70	22,000
Other Items	-	-	-	5,000	10,000,000

Individual values for the Halbach array given per m³

Due to lack of pricing and weight information, we have given a bulk estimate for other components to bring totals in line with Transrapid figures

COMMUNICATION

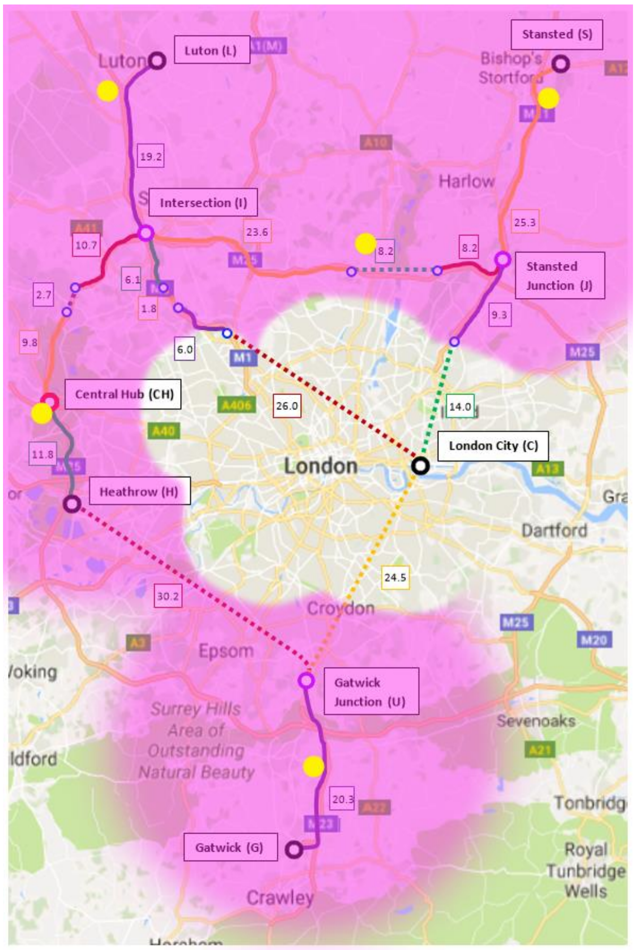
Since we have opted to use 2G frequencies for communication, we can use existing mobile networks to gain insight as to radio ranges. For overground antennae, there is little in the way of obstruction to the signal so ranges can be calculated using the Inverse Squared law, which states that the power of the signal will be inversely proportional to the square of the distance it has travelled. Of course, if there are hills/large buildings in the way, this will be reduced because there is not direct line of sight.



Topographic map of London. Source: http://en-gb.topographic-map.com/places/London-92172/

This topographic map shows the land height around London. Gatwick is located in the elevated area to the South and Luton is in the elevated area to the North-West and Stanstead to the North-East. Heathrow is in the lower area. In practice, 2G signals can reach 35km in open areas [48]. They can reach even further when not subject to the limitations of the mobile communication protocol, with distances of up to 250km. [49] Since we will be using more powerful, sensitive and lower noise transceivers in our maglevs, we anticipate a signal range of approximately 50km outside of London when line of sight is achieved.

Given this, overground network coverage can be achieved as follows:



Antennae are marked on the route map in yellow with their ranges in pink. All antennae have been placed in locations that either border the route or, in the case of the middle north tower, next to the M25. There is no extra greenfield usage for any communications systems. The southernmost antenna as well as the Luton and Stanstead antennae are all located in higher altitude areas which will give improved line of sight range whereas the other two are only needed for shorter distances. Finally, there is an overlap of at least 3km between all antennae to allow for communication hand over when a maglev transitions from one to another.

Underground communication is less simple because radio waves will be affected by the walls of the tunnel. All underground links are almost entirely straight lines, which will improve signal strength. In metro networks, radio has been successfully used underground with 1 mile transmitter spacings [50]. We shall aim to use spacings of 2km in our network, due to straight lines increasing the maximum communication range. This means we will require about 59 underground transmitters to cover the 118km of underground track.

ENERGY

Energy will be provided from the National grid. Currently the UK has limited power reserves and the grid may not be able to cope with the additional load from a high-speed maglev network. Expansion plans due to be completed in the mid-2020s should however provide a solution [51]. 16GW of additional nuclear power has been proposed to solve the energy crisis. The fact that the energy in this solution is nuclear is good, because there is an abundance of fuel available (even if it is not a renewable resource) and the nuclear waste is easier to contain and dispose of than greenhouse gases.

Analysis of required energy, as well as required velocities, on the track, is given in {8}.

TRACK DESIGN

GENERAL DESCRIPTION

The track width will be wide gauge, or 1600 mm, as a wider gauge reduces the possibility of derailing, and the body of the vehicle is quite wide. Our design's rail profile is the same as a standard Irish railway, and the tracks are made from steel, which is the standard material for modern rails [52]. The rail allows for the auxiliary wheels to guide the train when it is not levitating.

The track we will use is ballastless slab track. The base of the track can be cast out of prestressed concrete, which is a form of structural concrete that is pre-exposed to internal stress during the setting process. This is superior to normal structural concrete as it can withstand more stress and is more rigid in comparison [53]. These concrete sleepers both hold up the rails and dissipate the pressure from the weight of the train. The rail will be continuously welded (CWR, [54]) to provide a smoother ride when the vehicle is not levitating, and the stressed before installation, to prevent derailing from deformation caused by thermal expansion [55]. The rail body will be fastened to the concrete sleepers by Pandrol e-clips [56].

Embedded onto the surface of the concrete beside the rails are the levitation coils. These are closely-packed coils of copper wire, which gaps in between to allow room for thermal expansion.

There are two slopes running from the levitation coils to the two sides of the sleeper, which also have electromagnets built in. These are guidance coils that repel the secondary coils on the train in order to stabilise the vehicle. They are also used as transformers to charge the batteries on the vehicle when the train is at rest.

Both sets of coils are powered by a cable which runs alongside the track, that is connected to the National Grid at regular intervals. Another cable for signalling also runs with the track. This cable is connected to the sensors in the levitation coil, which detect the magnetic field of the permanent magnets on the train, allowing feedback to be given to the control system in order to update the powered sections of the track.

The width of the sleepers is 4000 mm, which is 450 mm wider than the width of the train. The gap between the sleepers is 1550 mm wide, which leaves the distance between two trains traveling in the opposite direction at 2000 mm. This gap allows the relief of air pressure when two trains are crossing on opposite sides of the tracks at high speed. The train turns by a slight kant in the track, and the component of gravity that is parallel to the slope of the track provides the torque which turns the train.

INTERSECTIONS & SWITCHING

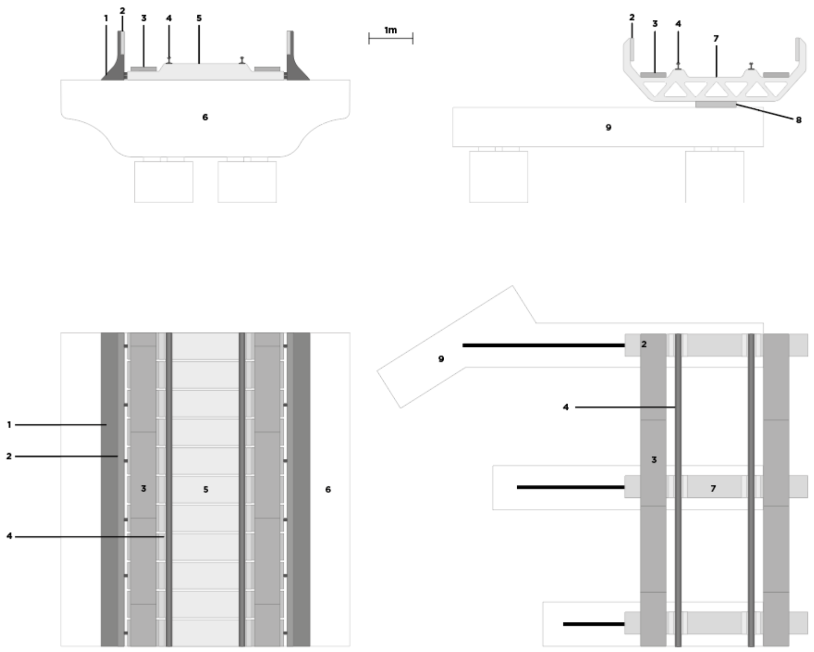
Regular HSR track switching is impossible with our design, as rails for one direction will always be in the way of the lower parts of the trains when moving in the other direction.

Instead, track switching is achieved by bendable switches. These are sections of track that can be moved perpendicular to the direction of travel by a group of electromagnetic setting drives. The rails and coils bend as each section of the track offsets to one side, and curves to provide a smooth turn, which allows the train to switch between one of two tracks. This is similar to the track switching system employed by the Transrapid system.

The switching is controlled by the computer system. After switching, the altered track is firmly secured in place by hydraulic locks, which minimises the risk of derailing. The switch track section is fastened to a set of frames connected to the electromagnetic drives instead of concrete sleeper, which both fixes the rails and coils in parallel, and bends them when the frames are offset by the drives.

The position of the bendable section of the track when it has turned (its "turnout" position) can be modelled as an arc of a circle whose radius is the minimum track turning radius, as calculated in {1}.

Dimensions of track switching sections are calculated in {9}.



1:Guidance Coil Support 2:Guidance Coil 3:Levitation Coil 4:Rail 5:Concrete Sleeper 6:Supporting Structure 7:Steel Frame 8:Electromagnetic Setting Drive 9:Supporting Structure with Groove for EM Setting Drives

UNDERGROUND

CONCERNS ABOUT TUNNELS

Our design requires multiple tunnels, ranging in length from 1.8km to 30.2km. Having a train enter a tunnel at over 300km/h presents several issues [57], which are discussed below.

MICRO-PRESSURE WAVES

Also known as sonic booms, micropressure waves can occur if the tunnel is designed unfavourably. Air in tunnels cannot escape like it does in open air, and is instead forced to move around the train when it enters. The train acts like a piston on entering the tunnel, pushing a large mass of air forwards and thus increasing its pressure. A pressure wave is formed that can steepen as it moves through the tunnel. Like any (non-surface) compression wave, it moves at the speed of sound, which in air is approximately 340 ms^{-1}. It therefore reaches the exit portal much faster than the train. On hitting the exit portal, the high-pressure air is partially reflected and partly radiated outside, and in extreme cases the exiting micro-pressure wave can detonate with a loud noise. Another (more moderate) micro-pressure wave can form when the train exits the tunnel.

These waves present multiple issues. A sonic boom would be unacceptable to nearby residents, but even if the intensity of the wave is lower, vibrations of doors, windows and walls is a large issue that should be mitigated as much as possible. The amplitude and gradient of the wave decrease inversely to the distance from the exit portal.

The pressure wave amplitude is proportional to train speed squared, and the gradient to train speed cubed, meaning that even moderate speed increases can cause significant problems when the train is already moving at a high speed [58]. The intensity of these waves is related to the pressure gradient of the incoming wave that can either become steeper or smoother during propagation down the tunnel, depending on the tunnel's design. Since some of our tunnels are extremely long (30.2 km), we must aim to smoothen pressure waves, since practically any steepening effect at all will accumulate to cause an unacceptable sonic boom upon exit.

There are no official standards for maximum acceptable micro-pressure wave amplitudes. The unofficial "Japanese criterion" states that waves are acceptable if below 20 Pa at 20 m and 45° from the tunnel exit [59]. However, this is unnecessarily conservative and has varying interpretations in literature. In Germany, it is required that the nearest dwelling should experience less than 70 dB (C-weighted), and at 25 m from the exit, the amplitude should be less than 115 dB (C) [58]. These regulations address shortcomings in other criteria, protecting people from dangerous sound near the portal with the latter, while preventing civil disturbance with the former. In fact, experience from other railways has shown that waves lower than 70 dB (C) are barely audible. As a result, since there is no strict legislation in the UK on the matter, we have inferred that it is acceptable for us to exceed this limit by a few dB (C). Therefore, if we aim for 70 dB (C) at 25 m from the exit, minor increases in amplitude due to random effects such as ambient temperature and pressure changes will not matter.

The pressure wave amplitude and gradient can be found approximately from calculations given in [60]. This analysis is described in {10} and is discussed in the next section.

PRESSURE COMFORT

Sudden changes in pressure can cause discomfort or even for the ears of passengers and staff. There are strict international criteria [59] regarding pressure comfort.

AIR RESISTANCE

There is a large amount of work done by the train in pushing air down the tunnel. This slows the train down and requires lots of additional energy to be supplied to the train. This is discussed in the section concerning optimisation of the train trajectories.

OTHER CONCERNS

In addition to pressure, other major concerns about tunnels include:

- Methods of dealing with crashes and fires
- The ability to evacuate from any point along the length of the tunnel

TUNNEL DESIGN

SINGLE VS. DUAL TUBE

The tunnels can either be designed as two tubes each with a single one-way track, or a single tube with a double track. There are benefits and drawbacks to both systems [61]:

Key: 0 means no effect, and the higher the number, the more positive the effect.

Parameter	**Single Tube**	**Dual Tube**
Air Resistance	**+2** Almost always one parallel free track, giving plenty of free cross-sectional area.	**-2** Much smaller free cross-sectional surface area, and more friction with tunnel surface.
Pressure Waves	**+1**	**-1**

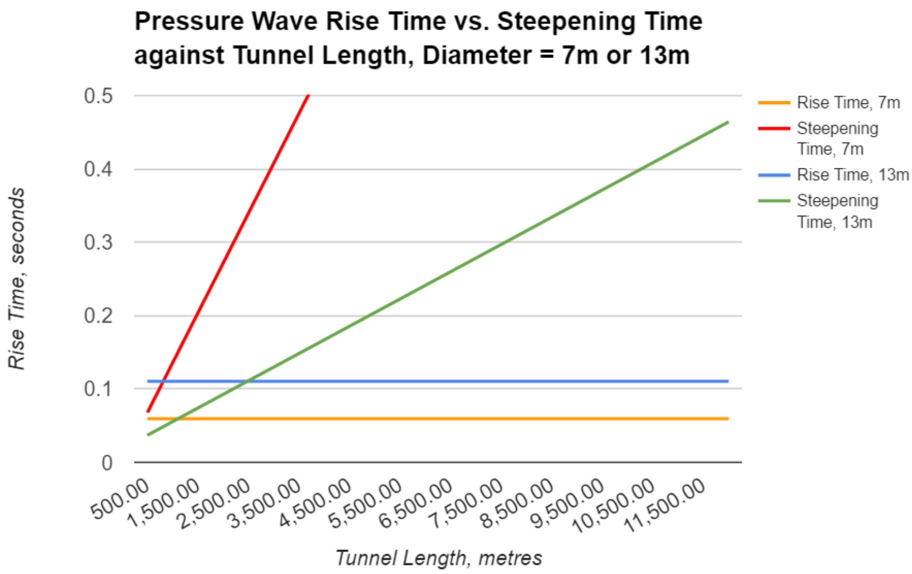
	Larger exit portal increases micro-pressure wave emission radius [5], lowering their amplitude. Larger cross-section for air escape increases pressure wave build time.	Smaller emission radius; increased pressure wave amplitude. Smaller cross section; reduced build time.
Pressure Comfort	**+1** (Long-term, for length of tunnel) **0** (Peak pressure wave magnitude) – although entry wave is reduced, when trains pass in opposite directions, a similar pressure wave peak is attained.	**-1** (Long-term, for length of tunnel) **0** (Peak pressure wave magnitude)
Cost	**+1** Less expensive per kilometre.	**-1** More expensive per kilometre.
Tunnel Climate	**+1** More heat capacity; air exchange due to opposite-moving trains.	**0** Acceptable air exchange.
Maintenance	**-1** Tracks in both directions must be shut down to perform maintenance on one.	**+1** Maintenance can be performed on one tunnel while keeping the other in operation.
Safety	**-2** Difficult to evacuate in case of emergency, although it takes longer for smoke to fill the tunnel in case of fire. Derailing is a danger for the other train; possible collisions.	**+3** Cross-tunnels allow regular escape routes into the other tunnel. If there is an issue on one track, it will not affect the other, which can continue to operate normally.

Although a single-tube system is superior in many aspects, safety is paramount in the construction of rail tunnels. Most modern high-speed systems are designed as dual-tube systems for this reason, notably Crossrail. Therefore, we will implement dual-tube tunnels for all long tunnels. For short tunnels, escape is easier and few escape routes would have to be constructed. It is not worth constructing dual tubes for these, especially when considering the associated additional air resistance, so the 1.8km and 2.7km North-Western tunnels will be single tubes.

MEASURES TO REDUCE PRESSURE BUILD-UP

Calculations from {10} show that the maximum pressure wave amplitude at 20m will not exceed 16Pa, or approximately 118dB {11}, which satisfies the Japanese criterion. At 25m it drops to about 116dB. The dominant frequency of the pressure wave can be approximated as V/L = 5Hz [58], and with C-level conversion attenuation the sound level can also be expected to satisfy the German criteria.

Below is a graph using calculations from {10}, showing the pressure wave rise time and steepening time for approximate single-tube and double-tube tunnel diameters:



For the majority of tunnel lengths, the pressure wave steepening time is longer than the pressure wave rise time, implying there will be a detonation at some point. This obviously presents an issue. Luckily, pressure wave damping is in fact easier with longer tunnels. This is because as long as the pressure wave reduces in gradient over time, it will have sufficient time to die down to negligible amplitude by the end of the tunnel. We simply need to create a tunnel environment which dampens, rather than steepens, the pressure wave as it moves through the tunnel.

There are several possible measures to allow this [58]:

Measure	**Discussion**
Holes within cross-passages to allow air flow from one tube to the other	No design guidance or assessment methods could be found. Unfavourable timings of trains moving in opposite directions could perhaps steepen, rather than dampen, pressure waves. Investigation would therefore be required. If holes are too large, there is a risk of smoke spreading to the neighbouring tunnel in case of fire. This could be dangerous for evacuation.
Ballast	Ballast is well-known to reduce pressure build up, especially in long tunnels. Even with the slab-track necessitated by TBMs (see next section), ballast can be added on top. However, the cross-sectional area of the tunnel would be

	slightly reduced, possibly counteracting benefits, and decreasing pressure comfort. In addition, no reliable method is available to calculate the required quantities of ballast.
Acoustic absorbers	Nakao et al [62] have performed scale model experiments with porous resin occupying 7-21% of the tunnel cross-sectional area. At 7% wave steepening was prevented, and at 20% the gradient reduced by a factor of 10-20 when the tunnel length was 600 times its diameter.
Active Cancellation	Ozawa et al [63] tested scale models using "anti-sound" active noise cancellation played within the tunnel through loudspeakers. These were successful. This would be a low-cost method, but is subject to electrical or calibration errors, and would require constant maintenance.
Speed Restrictions	Although speed could be reduced slightly, it would not reverse pressure wave build-up, and high speed is of course required to meet the goal of 20 minute journeys.
Air Spaces	Small holes can be punctured in the main tunnel leading to free air chambers, as discussed theoretically by Sugitomo [64]. There is no experimental evidence showing this has benefits, but it would not cost a lot to use existing areas of tunnel cross-section underneath walkways and the track.

A further list of measures relating to tunnel shape is given in [57], however, few are easily applicable to our project, although the benefit of open cross-passages is reinforced by P. Reinke et al. in [61].

Due to strong evidence in its favour and relatively simple installation, we will use porous resin acoustic damping in our longer tunnels to smoothen pressure waves. It will cover 8% of the tunnel cross-section. Other methods are either too uncertain or could have potential, uninvestigated drawbacks.

This model is more suitable to show how we will handle the shorter tunnels, in which acoustic damping will not have sufficient time to reduce the pressure wave gradient by the time the wave reaches the exit portal. For short tunnels, the simplest way to minimise pressure wave amplitude is to build an entry hood.

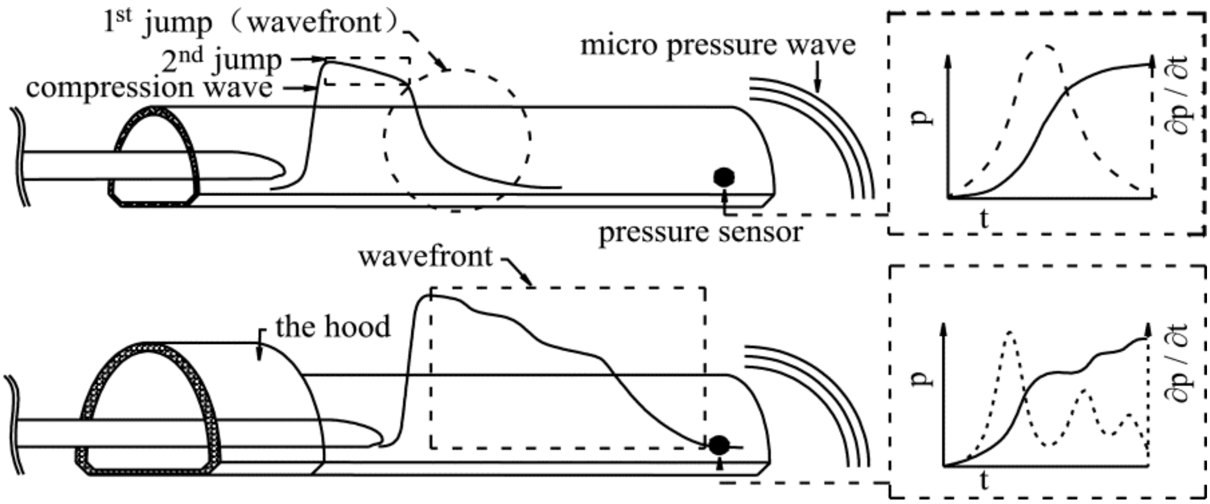
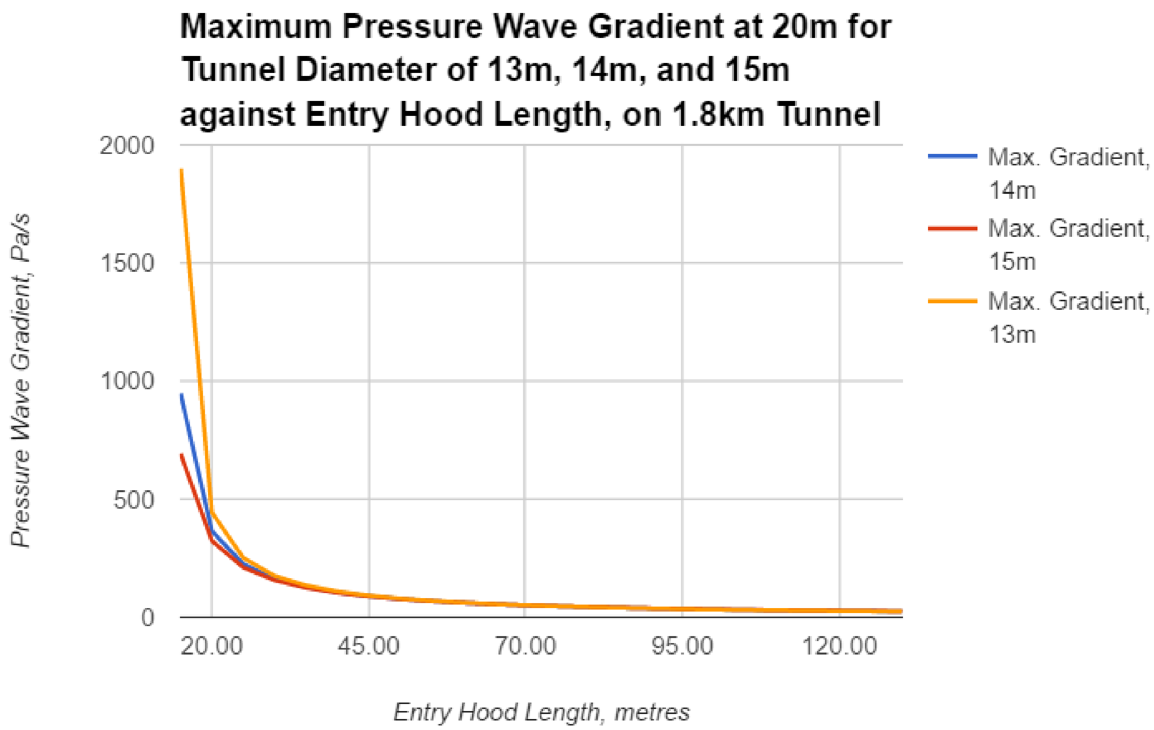
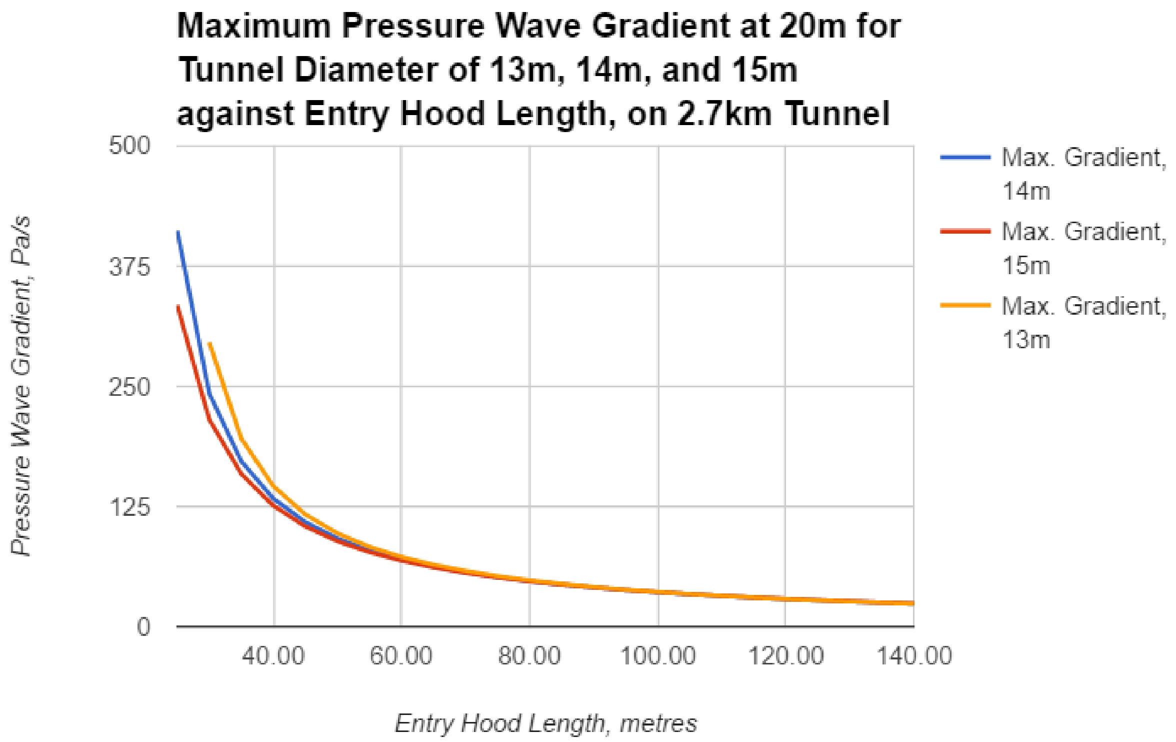


Diagram of the Effects of an Entry Hood, source: Xintao Xiang et al. [65]

There is space at both ends of short tunnels to install these, although attention would have to be paid to make them attractive and not too much of an eyesore. These graphs indicate the maximum amplitude at 20m from the exit portal for various tunnel diameters:





It is clear that increasing tunnel diameter has almost no effect past a moderate entry hood length, although it causes large relative benefits for short hoods. Since it is the least expensive and most structurally stable, we will therefore use a 13 m tunnel diameter. This requires an average thickness across the tunnel wall of about 27 cm of porous resin. One major disadvantage of this is the increased blockage ratio and therefore wasted energy used to do work against air resistance. However, the blockage ratio is still small due to the fact that the short tunnels are single-tube. On balance, the reduced cost of tunnel construction outweighs the cost of the extra energy used.

It is difficult to predict what the true pressure wave gradients will be. The model is simplified, and acoustic damping will certainly have some effect. Luckily, entry hoods are simple to install. This means that we will be able to test the pressure wave amplitudes and gradients on the real system once it is constructed, and install entry hoods only if necessary and to more accurate specifications.

Short tunnels will have a diameter of 7 m, as this is large enough easily to house the trains and provide moderate cross-section for air escape as they pass, whilst not being so large as to be needlessly expensive.

SAFETY MEASURES

There are comprehensive guidelines that should be adhered to when designing railway tunnels. A sensible set of standards is put forward in [66], and guidelines for multiple countries including the UK specifically on fire safety are compiled in [67]. The following sections comply to UK specifications and provide additional safety features.

In all tunnels, there will be linear heat detectors installed, as well as fire alarms and smoke detectors every 100 metres. Additionally, there will be safety niches every 150 metres, with a sliding door and dimensions 2.2 x 2.5 x 1.5 m (height x length x depth).

LONGER TUNNELS

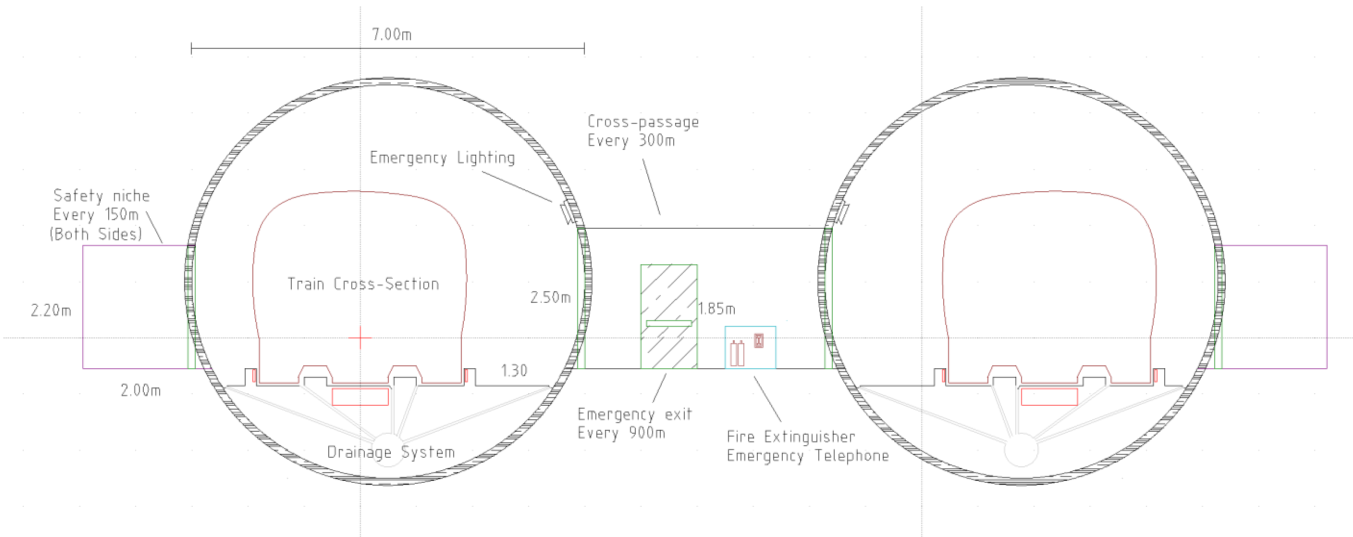
Firstly, cross-tunnels are required every 300 metres. These will have sliding doors on either end that are normally shut (sliding doors withstand pressure waves better than regular doors). The doors will seal the tunnels from each other.

Every three cross-tunnels will have an emergency staircase to allow passengers to evacuate in case of emergency. This will have its own sliding door and will be perpendicular to the cross-tunnel. The staircases will reach the surface at convenient positions close to the cross-passages.

Every cross-tunnel will have fire-fighting hydrants/extinguishers and an emergency telephone.

An emergency walkway 1.20 metres wide will be provided on the inner side of each tunnel. The walkways will have a handrail and emergency lighting, which will only come on when required. They will have narrow open drainage channels by the wall for removal of groundwater running down the tunnel walls. A drainage system will also be installed with pipes underneath the tracks.

For regular operation, natural ventilation and the piston effect of trains moving through the tunnels will suffice for tunnel ventilation. If a fire occurs, forced longitudinal ventilation in the direction of travel of the trains will be applied. This allows trains downstream to escape out of the tunnel unobstructed, while people evacuating can move in the opposite direction, clear of smoke. The critical velocity required to push smoke so that no backlayering can occur is normally 2.5-3 m/s, so ventilation which can push air at 3 m/s is safe. This destroys stratification of smoke, but the ability of people to be swiftly be able to move upstream of it and have no smoke at all spreading towards them outweighs any negative impact.



SHORTER TUNNELS

The safety features will be very similar in the 1.8km and 2.7km tunnels. However, there will be some differences due to the single-tube, bidirectional tunnels.

Every 300 meters, there will be side rooms with sliding doors. These will contain fire extinguishers and emergency telephones. There will be a similar emergency walkway on both sides of the tunnel. There will be a drainage system with pipes between the tracks.

Longitudinal ventilation will also be installed, but operation and evacuation strategy will have to be decided contextually.

FIRE RESPONSE [68]

As discussed, the tunnels will be ventilated to allow safe escape.

In the long tunnels, ventilation will be controlled automatically, because the response will always simply be to push air downstream of the fire site, and automatic response is fast and therefore effective.

There will be a semi-automatic control of the ventilation system in the short tunnels. The tunnel operator will choose the procedure of the smoke control system and start it. The control system will then control the ventilation according to a programmed procedure. This is more effective than manual control. Automatic control is not appropriate because the strategy of evacuation must be decided contextually.

An example sequence of events during a fire is:

1. Ignition
2. Communication to the tunnel control centre, from fire alarms, manually triggered and validated with smoke detectors and/or linear heat detectors. Possibly also triggered by emergency phone calls.
3. Evacuation, assisted or unassisted by emergency services
4. Simultaneously with 3, operation of ventilation
5. Firefighting

TUNNEL BORING

There are many methods of digging tunnels [69], but easily the most commonly used and most efficient for rail tunnels is the use of Tunnel Boring Machines (TBMs) which are large worm-like machines that slowly make their way forwards, cutting into rock and earth ahead with a range of head types.

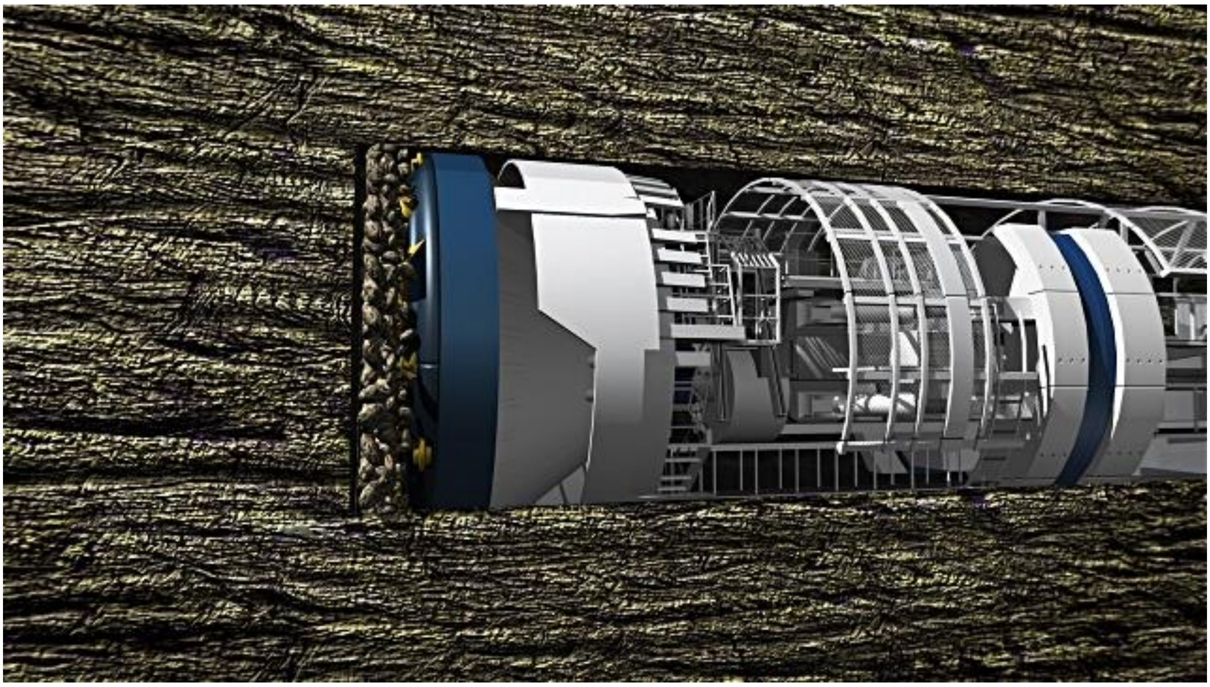
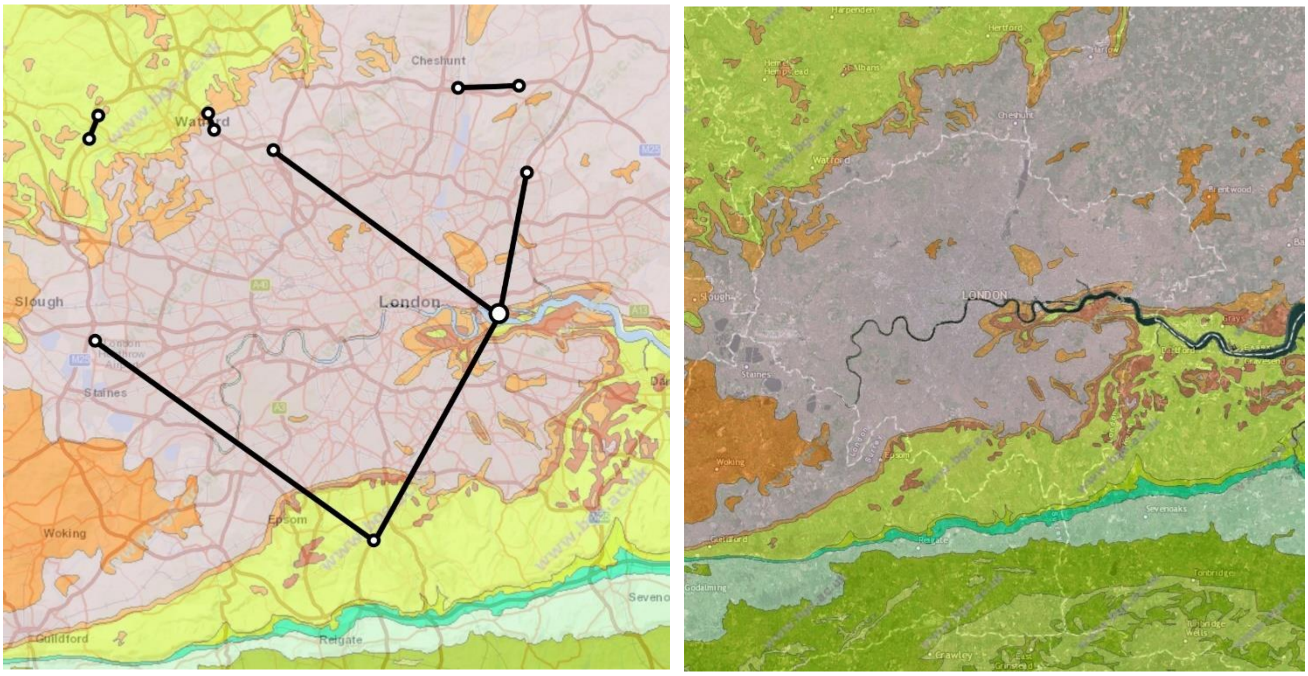


Image source: [70]

The first thing that must be known for the boring of tunnels is the general geology of rock to be tunnelled through. Here are maps with bedrock data from the British Geological Survey webpage [71]:



Left: Bedrock map with tunnels and roads, Right: Bedrock map showing topology of area.

Key (In order of coverage of tunnels):

	Thames Group - Clay, Silt, Sand and Gravel
	Chalk
	Bracklesham Group - Sand, Silt and Clay
	Thanet Formation - Sand, Silt and Clay
	Black Lines - Tunnels

The majority of tunnelling occurs in Thames Group bedrock, which is composed of clay, silt, sand and gravel. The 2.7km North-Western tunnel and the bottom segments of the Gatwick connection tunnels are situated in chalk. Short segments of tunnels are in bedrock composed of sand, silt and clay.

There are three main types of ground that TBMs operate in [72], [73]:

1. soft ground
2. heterogeneous ground
3. hard rock

Clay, silt, sand and gravel all fall into the first category, while chalk is a very soft rock [74] that comes under "heterogeneous ground". This implies that the majority of tunnelling should be performed by "Earth Pressure Balance" (EPB) tunnelling machines, which work in soft ground, while the remainder should be dug by "Slurry" or "Mixed-Shield" machines. Both of these varieties can dig tunnels both with diameter 7 metres and 14 metres.

Before final decisions about TBMs can be made, test bore holes will have to be made along the routes to gain more detailed knowledge about ground conditions, including specific bedrock type and water content. Crossrail dug 1906 exploratory boreholes over a period of 2 years before construction [75].

CIVIL CONCERNS

Digging underneath people's homes is only an issue when dewatering is required. This presents the concern of ground settlement, which can lead to damage to buildings. However, on Crossrail's project, there was very little effect on any buildings, despite large-scale dewatering. A small number of plaster ceilings collapsed, but the stability of these was not known before the dig. An inspection can be done on houses above tunnelling routes for our project.

We will use Satellite Interferometry (InSAR) for monitoring, which can detect ground settlement down to a hundredth of a millimetre.

Another potential concern of dewatering is the risk of trapping air as water levels recover once pumps are turned off, which can then be forced into tunnel shafts. This problem will be fixed with venting.

COSTS

It is difficult to estimate tunnelling costs, particularly because TBM costs are only given on request by companies. We estimate costs based off data from other underground railway tunnelling data in the finance section.

OBSTRUCTIONS

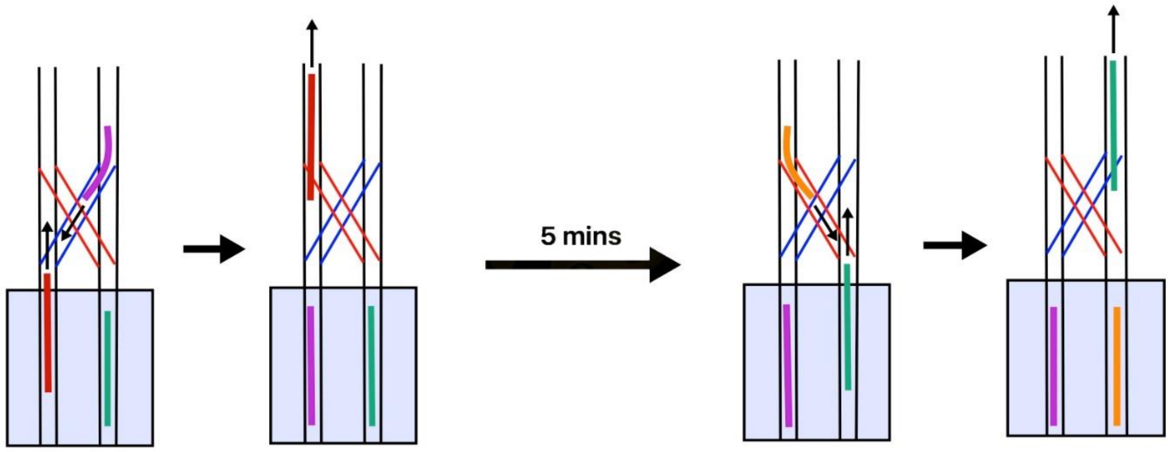
Underneath Central London there are already many tunnels. Our tunnels may have to diverge from their straight paths slightly to weave around these. This should not be a significant issue, because most trains will be travelling slowly near the City station. Trains going directly from Gatwick to Stansted may, however, have to slow slightly.

STATION DESIGN

PLATFORM LOGISTICS

Normally on railways, trains stop at various stations *en route* to a destination. This method would not work with our system due to the sheer speed required. The only real practical system is to have trains going back and forth between each pair of airports.

When addressing the design of our stations, the first main problem to deal with was the fact that our system must conform to the specification of a maximum waiting time of 5 minutes. Having a single platform for each connection would give each train only 5 minutes at a station before departing. In this time, all arriving passengers and their luggage would have to disembark the trains, and the next set of passengers would all have to get on, stowing their hand luggage and strapping into their seats. With up to around 340 passengers involved in the turnaround, attempting to do this simply isn't viable. Therefore, we have

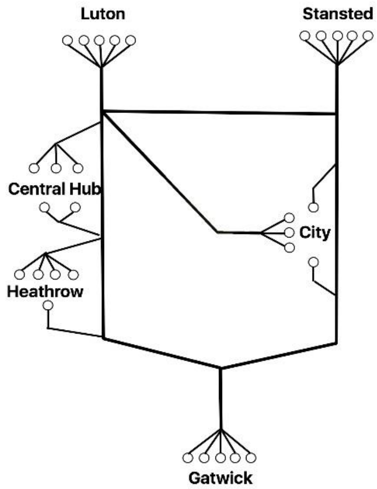


designed this solution for every connection:

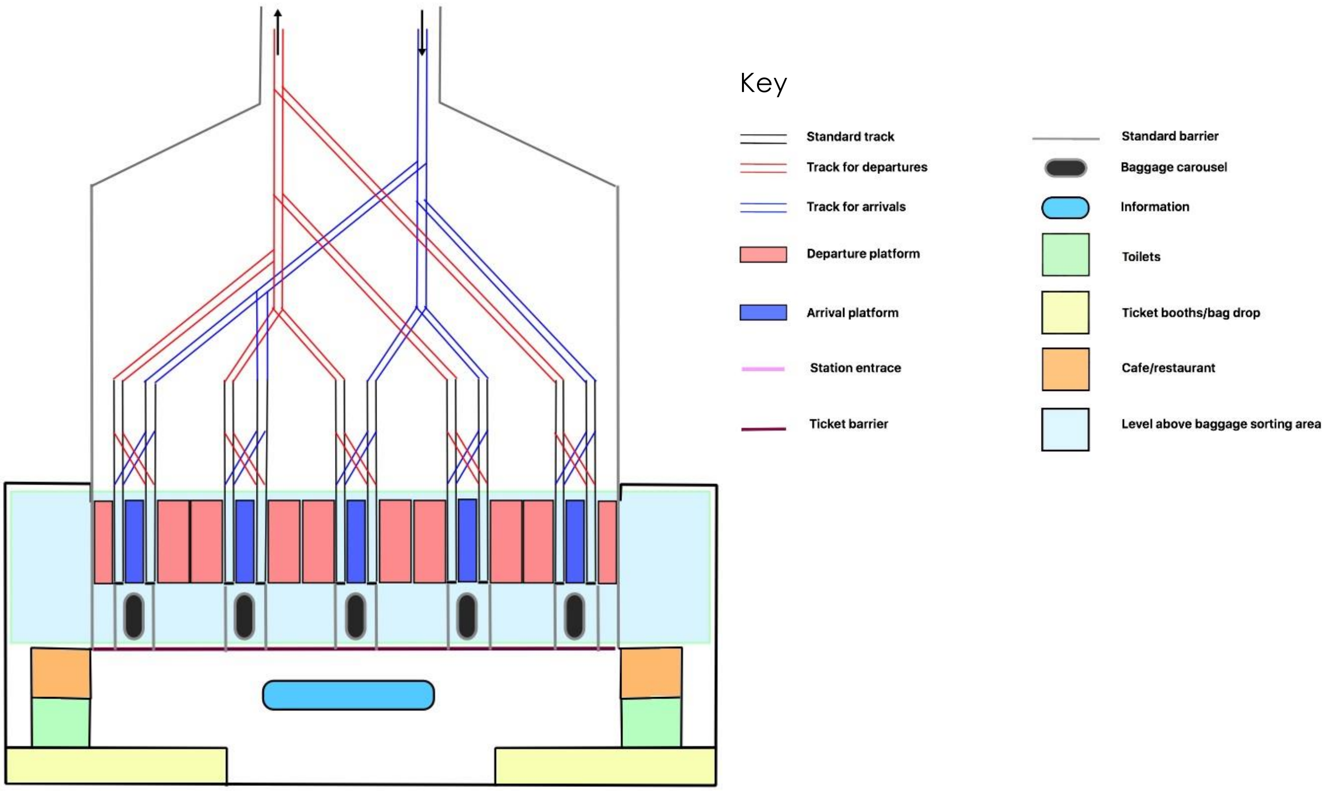
In this solution, there are 2 platforms dedicated to each destination at every station. This way, the rate of one train every 5 minutes (for each destination) is still achieved, however, each train stays on the platform for 10 minutes rather than 5, allowing a much more realistic time for passenger turnover.

TRACK CONNECTIONS & STATION PLANS

Luton, Stansted and Gatwick are located on the ends of their lines, and so the main lines start directly from those stations. However, Heathrow, Central Hub and City are located on top of main lines with trains travelling at up to 360 km/h. Therefore, we have located the stations off to the side of the main lines, allowing the fast-moving trains to bypass the stations with no safety concerns of passengers being near high speed trains. Slip rails (similar to motorway slip roads) will come on and off the main line and connect to the hubs, maximising safety because they make sure trains are up to speed when they join onto the main lines which means sufficient distance is maintained between trains already on the line and trains which have just joined. There is a bridge over the main line for slip rails on the opposite side to the stations.

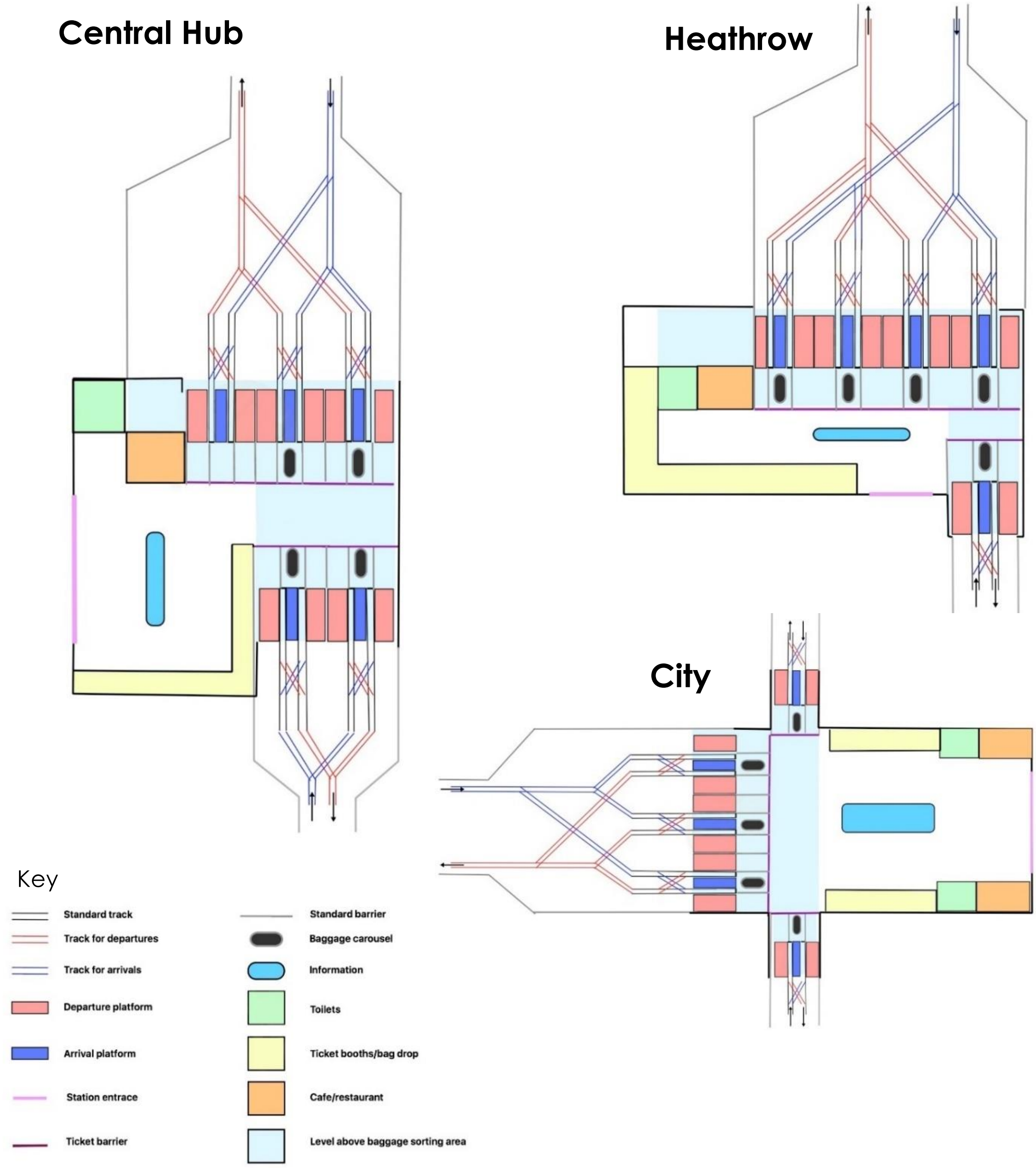


The stations at the end of their lines (Luton, Stansted and Gatwick) were fairly straightforward to design because all arrivals and departures are in the same direction. This is the design of our standard end-of-line station:



Heathrow station and the Central Hub are located on top of the main western overground line. They have a very similar design to the standard end-of-line station, however, since trains need to travel from opposite sides of them, they will have to be adjusted slightly.

The London City airport will have further differences from the others, because it is the only underground station in our design (due to lack of overground space and the fact that all the lines meeting it are underground). It is at the end of one line from the West connecting it to Heathrow, Central Hub and Luton, and on top of a North-South line with the Stansted connection coming in from the North and the Gatwick connection coming in from the South.



STATION OPERATION & SERVICES

Departing passengers enter the station, buy their tickets and (if commuters) drop off their checked luggage at the ticket booth and bag drop areas. Having these facilities on both sides of the station will spread out the passengers, minimising the time spent in the station and allowing quick travel to the platform. Dropped-off bags will be taken up to the level above the platforms via a conveyor belt and put in a container to go to the correct destination hub. There will be a luggage tracking system (explained in more detailed in the next section) which will give passengers information of the whereabouts of their luggage at any point in time.

The passengers use their tickets to access the correct departure platform for their train. Having specific ticket barriers for each platform stops passengers from attempting to buy a cheaper ticket and then embarking a train that should have cost more to travel on. Having embarked their train, passengers stow hand-luggage and sit in their seat, ready for departure.

Arriving passengers disembark onto the arrivals platform. Those about to leave the airport system can collect their checked luggage on the carousel between the arrivals platform and the ticket gate. Their checked luggage will have been sorted in the sorting area and carried up to the carousel for passengers to collect. Having the specific carousel from each hub within that particular arrivals area minimises the danger of people accidentally taking others' bags, as well as speeding the retrieval process.

Any enquiries can be dealt with by an information desk located in the centre of the station. There are extensive toilet facilities here (as trains don't have toilets). Food and drink is also available from the cafés/restaurants in the stations, allowing passengers and anyone picking up or dropping off to have refreshments available, as well as generating additional profit for our system.

All of our stations are easily accessible, as explained earlier in the planning and environment section. To increase ease of the short journey between the station and its airport terminal where necessary, there are travellators between the station and terminal (blue lines on the Google Map). These travellators are three metres wide, avoiding the possibility of congestion, and go in both directions. They are sheltered by a clear hood that cover the sides as well as the top that allows natural light in as well as, of course, preventing rain getting in.

BAGGAGE SYSTEM

Luggage is essential for travellers and therefore needs to be handled efficiently and safely.

The extension of the time spent by trains at stations (from 5 minutes to 10 minutes) significantly reduces the chance of mishandling luggage and safety issues. However, incorporating a baggage loading system into the train system is still a challenge. This is due to a combination of factors:

- Most aircraft passengers take more than one bag with them on journeys. This increases the bag-to-person ratio from that experienced on regular railways.
- In order to allow airlines to coordinate their flights and make transfers convenient, passengers must be able to check in their checked bags at their source airport, rather than directly take it with them on the train.
- Taking checked luggage from passengers so they don't have to personally bring them on and off the trains is necessary to realistically meet the 10-minute time constraint of the unloading and reloading process.
- The high-speed nature of the trains provides a risk of movements and potentially sudden impacts inside carriages. This could cause heavy luggage to injure passengers.

It is clear from these considerations that luggage should be kept separately from passengers. The process of loading must also be integrated into airports' existing baggage-handling networks.

Another advantage of this is that luggage is packed more efficiently into the trains, maximising passenger space and comfort.

ULD CONTAINERS

We have decided to transport luggage in ULD (Unit Load Device) containers. These containers are normally used to load luggage onto aircraft. They allow a large quantity of cargo to be bundled into a single compact space [76]. Each ULD container will be transported on and off the train via crane lifting. Because this requires fewer units to be loaded, it saves time and effort and it helps prevent train delays

There are many types of unit load containers, but the LD-1 variety has been selected for this baggage loading system. This container type has the following features [77]:

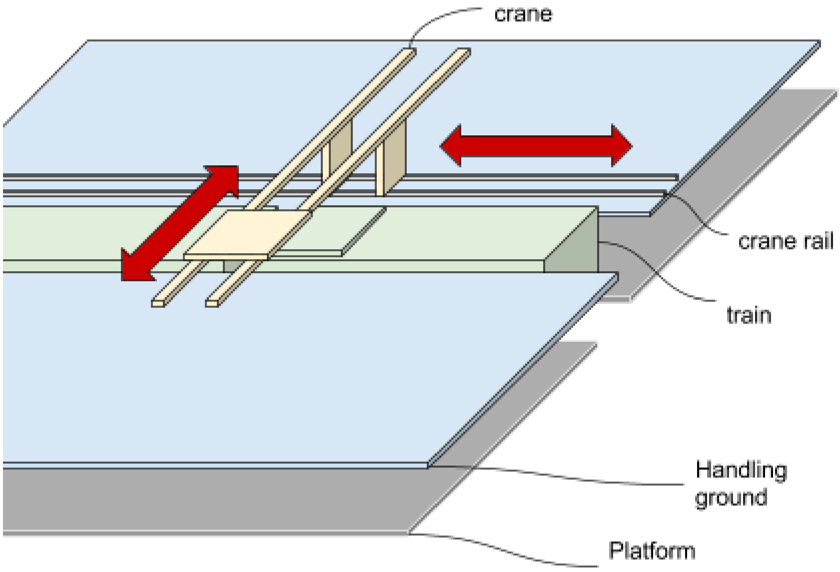
- Maximum gross weight of 1,588 kg
- Weight when empty of 70 to 170 kg
- Volume: 5.0 m^3

There will be two of these containers in each passenger luggage section. This will provide 10.0 m^3 of space for baggage per carriage, which is sufficient to carry every passenger's luggage {12}.

STATION (2ND FLOOR) HANDLING GROUND

On the roof of the luggage section of the train, there will be a sliding door. The luggage container will be lifted through this and transported by a crane up to a handling ground. The handling ground provides space for distributing bags to their correct trains.

There will be one crane for each train stopping area. The crane will be able to move parallel to the sides of the train, to align its position to the location of the luggage container. When one of the containers is attached to the crane, it will then move on the horizontal axis perpendicular to the crane rail and place the empty container back through the sliding door.



PASSENGER TYPES

- Regular commuters will drop off their bags at the departing station, and pick them up at the arriving station. The luggage needs to be on the same train with the passengers. This is because it is inconvenient for commuters if they have to wait another 15~20 minutes for their bags to arrive after they have arrived. It is very unlikely for regular commuters to have bags; thus, it is easy for their bags to be set as priority during the loading/unloading process.

- Arrival passengers are passengers who have got off a flight and need to get to a different airport as part of their journey home. In this case, passengers need to register their luggage to the train system in their departing airport check-in counter. There, they will be given a QR code sticker(tag) on their luggage. This QR code will be used when the luggage arrives in one of the London hubs. The airport handling system will send the luggage with the QR codes to the station and the luggage will be delivered to the destination station. This also applies to transfer passengers.

- Departing passengers are passengers who need to get to a different airport to catch their planes. They will check-in their luggage in their nearest station, where a tag will be applied. The luggage will be transported through the airport's baggage-handling system to the train station, where it will be loaded onto a train to the correct airport when space is available. The timing of this train does not matter, as long as the bag reaches the plane before loading ends.

LUGGAGE TRACKING SYSTEM

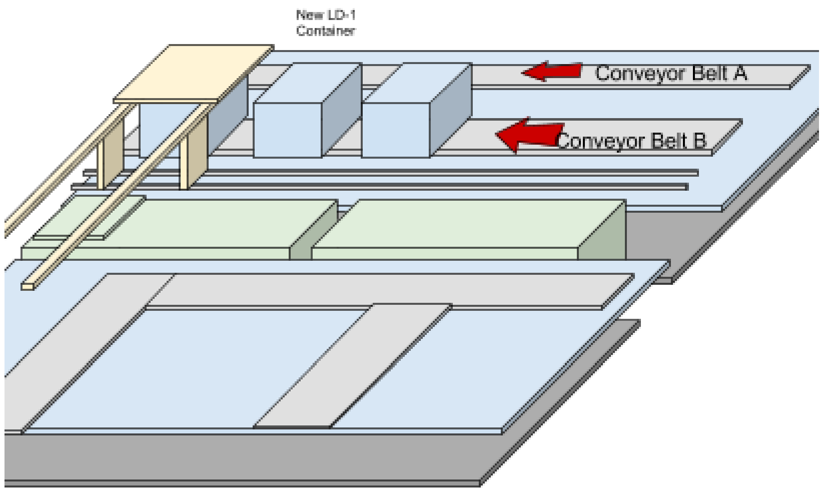
A sticker tag attached to each piece of luggage, even those from the source airport baggage-handling system, to ensure compatibility between airports. The tag will have a QR code stating several pieces of information:

- Passenger destination and departing station
- Passenger contact details
- (If applicable) Passenger flight details
- Expected time of arrival

Checkpoints within the station will be able to alert users of the system to the location of their luggage. It will include estimated time of arrival at the destination station. A phone app and text notification service will also available. The tracking system will prove particularly useful to those who have checked in their luggage.

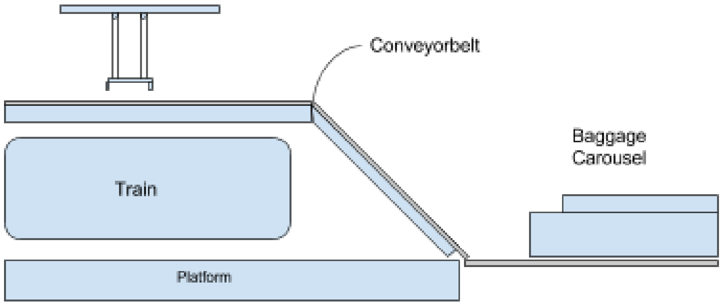
LOADING PROCESS

There are two sources of luggage during the loading process, one from the airport handling system and the other from the station. The luggage from the airport handling system will be delivered to the station through interconnected conveyor belts which extend to conveyor belt A.



Conveyor belt A will also transport luggage dropped off at the station to the baggage-handling ground. There will be separate baggage drops for each route. Another, separate conveyor belt will be used to transport luggage the two sources to the handling ground.

When the luggage arrives in the handling ground, it will be manually packed into ULD containers by workers. The containers will be queued on conveyor belt B. The loading process will start once the recently-arrived luggage containers are safely transported out from the train. When the unloading process is done, the cranes will load the new containers in the train from the end of conveyor belt B.

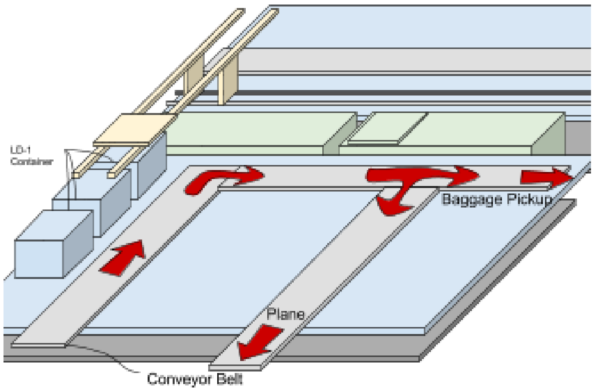


UNLOADING PROCESS

After the luggage is safely out of the train, the luggage containers are placed on to the unloading dock, where an automated container unloader unloads the containers with minimal human intervention. Lack of human contact minimises chances of baggage loss.

In the unloading dock, the LD-1 containers are gradually tilted, door facing the ground. As the LD-1 container gets tilted, the luggage in the container will slowly come out to the conveyor belt.

Luggage is than sorted onto one of two conveyor belts based on a scan of its tag, one of which leads to the carousel in the station, while the other takes bags to the airport's handling system.



A maximum of 10 minutes to safely move the luggage out of the train and replace it with new luggage. A maximum of about 40 seconds to take one container out giving a maximum of 4 minutes to move all 6 containers out of a three-carriage train. Up to another 4 minutes are then taken to move new containers onto the train. There are then two minutes left over for a final check.

RELIABILITY & SAFETY

The aim of this section is to ensure the network will have the procedures and technology in place to still operate effectively in the case of a sudden event such as a breakdown or accident, and also in the case of longer maintenance and upgrading periods.

Making the network as safe for passengers and operators as possible is vital.

MINIMISING BREAKDOWN FREQUENCY

Disruption occurs as a result of three main failures:

1. Technological failure of the train, such as an electrical fault or corrosion of a mechanical component
2. A failure of the line
    - Tree branches etc. may fall and obstruct the line.
    - Power failure e.g. a short circuit.
3. Unpredictable intervention
    - Human trespass on the line
    - Crash
    - Other – for example last year a Singapore line was disrupted when a commuter broke open a train window after the air conditioning failed.

It is clear that the majority of disruption occurs because of either minor incidents (fallen branch etc.) or major incidents resulting from one failure leading onto another failure (such as a trespasser causing an electrical fault).

The former type of failures can be easily minimised by well-thought-through design and proper maintenance, the procedure for which will be discussed. For the later, more serious events, larger scale redundancy will be discussed.

Overall, our system will have in place an operational code that ensures our network meets the high demands placed on it efficiently and safely.

REDUNDANCY

There is redundancy built into our technology – this is a key aspect of our design, and discussed at relevant intervals throughout the engineering section.

We will also have three back-up diesel locomotives capable of running on our lines in case of multiple failure of locomotives.

Backup electrical generators will be present in the case of a loss of power from the national grid. Trains would immediately stop in this event, and be towed back to stations by diesel trains after passengers have evacuated if the power loss is prolonged.

MAINTENANCE PROCEDURES

Our trains will have regular maintenance checks, focusing on the safety systems and ensuring a high level of efficiency.

- Regular checks will take place at night time when our network is at its quietest. There is a 2-hour window in the middle of the night where the network is completely quiet (see scheduling).
- All trains will have a thorough check up and servicing on an annual basis

In addition to regular upkeep of the line, the following maintenance procedures will be enacted:

- Ensuring the line is properly fenced off with large barricades to prevent trespass. The track is also raised, adding further deterrence.
- Trimming of vegetation cover near the line.
- Regular testing and cleaning of signal electrical systems.
- Removal of snow/leaves from track in times of harsher weather.
- We will have one conventional train which monitors the condition of tracks by filming them with an attached camera and other measurement tools, much like the NMT for Network Rail. [78]

BREAKDOWN RESPONSE

Because of the steps taken to ensure redundancy in our technology, the likelihood of a major crash has been minimised. However, in the unlikely case of such an event, our solution will satisfy the following essential requirements:

- Quick and easy access for the emergency services: there will be a number of secure access points along the line. The dimensions of the tunnels will be such that entry to underground parts of the line is easily possible.
- An instantaneous shutdown of that section of line – our centralised control systems will enable us to manage the whole network to make this possible (see below).
- Removal of damaged/powerless trains from tracks – this will be achieved using the diesel locomotives. The maglevs will have wheels that fit the rails on the track, so if they lose power it is possible for the diesel locomotive to tow or shunt the maglev to a siding or back to the central hub.

The damaged train can be stored at our central command hub. There will be tracks allowing trains from any line to directly join onto the Central Hub line, so trains can easily get to/from the Central Hub in case of mechanical issues.

ROUTE REDUNDANCY

Should a section of the route between two stations be blocked, the following table summarises the diversions that would be made to allow passengers to make the key connections between Heathrow, London City, the central hub, and the Gatwick junction in the minimum time possible.

Blocked Route	Approximate distance/km	Diversion	Extra distance/km	Approximate Time Delay/minutes
A	35	Via B, D and C	57	10
B	30	Via A, D and C	73	12
C	25	Via B, A and D	79	13
D	40	Via E and F	49	8
E	23	Via D and F	53	9
F	36	Via E and D	27	5

For the network in general, if a diversion is possible the calculated time delay will never be more than 15 minutes. Note that there is redundancy for all routes except for out of Gatwick, Luton and Stansted.

The logistics of diversions are as follows:

- Diverted trains will operate in addition to the trains already scheduled to run as normal along the routes.
- This will be achieved by minimising the distance between trains and using the maximum number of carriages per locomotive.
- An average speed of 340km/h was used to calculate the table (slower than the normal average speed) because for safety reasons the trains will be slowed down slightly when carriage length is maximised and distance between trains is minimised.
- Where there are no direct routes, such as when **A** is blocked and passengers must go from **C** to **D** in order to reach Luton from Gatwick, they will have to get off the train at the station, then get on a separate train to continue their journey.

CENTRAL HUB

A key part of the running of the network is the central hub. This will be located on the intersection between the Heathrow-Luton route and the M40.

This is where the spare locomotives (diesel and maglev) will be stored, and where locomotives will be brought for maintenance and repair.

SUMMARY FOR RELIABILITY & SAFETY

A safe, reliable network is made possible by three key design points:

- Our technology has redundancy built into it.
- There are procedures in place for the event of a breakdown, and in the event of extended maintenance.
- Co-ordination from a central hub minimises inefficiencies in the running of our network and ensures there is accountability for safety and upkeep of the line.

FINANACE & IMPLEMENTATION

PROFIT ESTIMATION

MARKET RESEARCH

The first thing we need to calculate is the profit we will gain from our system. It is useful to begin by looking at competitive methods of getting from airport to airport.

The table below gives the approximate travel time averaged over all routes, as well as the price range for each mode of transport [79-82].

	Taxi	Bus	Train	Our Solution
Avg. Travel Time /mins	$\approx$60 Varies Significantly	$\approx$100 Varies Significantly	$\approx$100	≤ 20
Avg. Cost	£114.59 (£64.9 - £170.20)	£22.14 (£11 - £33)	£35.19 (£15.9 - £68.7)	£x
Avg. Cost * Time (/mins)	£6875.4	£2214.00	£3519	£20x
Above divided by 20	£343.77	£110.70	£175.95	£x
Issues	- Traffic dependent. - High-cost	- Traffic dependent - Sometimes unavailable - Difficult with bags	- Difficult with bags	

Taxis and buses are unreliable in that their travel times can vary significantly even on the same route, depending on traffic or road closures. Buses are also sometimes entirely unavailable. Existing train lines can suffer somewhat from reliability issues, but are much less likely to have delays than taxis or buses. The same applies for our solution.

Working on a basis of cost alone, the upper bound for our system to be competitive is about £115, and to beat all currently available options tickets must be less than £22 on average. A better metric is to weight price based on time, because a faster journey can be seen as more valuable. Using this gives a much more lenient upper bound of about £344, and a price to beat other available options of less than £110.

In theory, tickets could therefore be very expensive. Although we have been given fixed minimum average volumes of people taking our trains every hour, we should treat this as a realistic system in which high prices will dissuade potential customers. We should therefore attempt to minimise price within sensible constraints.

ALTERNATIVE REVENUE

There are multiple ways we can generate profit with our train system, aside from ticket sales.

ADVERTISING

An average billboard in a city costs in the region of £5,000 per month, or £60,000 per year [83]. If we optimistically say we can put up the equivalent of 200 billboards in the network, this would give us £12 million per year.

SNACKS

We can optimistically say we will sell £200 worth of food per hour per station. This would give around £9m per year.

DONATIONS

Crossrail received [84]:

- £70m from Heathrow, due to the benefit of improved links to the city
- £200m from the City of London Corporation
- £150m from the Canary Wharf Group, towards the costs of the Canary Wharf station.

We could optimistically expect a total of up to £250m of independent funding from the involved airports, due to the benefits discussed in the original proposition of the project.

SUMMARY

The sum of these contributions could contribute up to perhaps £700m over 20 years, and would probably contribute less. While this is a large sum of money, it is relatively insignificant against the quantity of money earned from ticket sales.

TICKET SALES

The total congestion figures are given in the appendix, in section {13}. We can use these to estimate profit from ticket sales.

There are several options for pricing tickets. We could have a set price for all routes, however, this would be inappropriate, as some routes are extremely short (for example, Heathrow to Central Hub) while some are extremely long (Luton to Stansted).

We could arbitrarily define prices for different routes, or we could define a price per kilometre, and base all prices on this value. It would be logical to have a price somewhat proportional to route length, so we chose this option. However, we weighted this price per kilometre based on route length. Details are given in {14}. We rounded the route lengths to the nearest 5km before calculating ticket cost, to avoid a situation with prices that are precise down to a tenth of a penny.

As previously mentioned, in the highly-congested peak periods, it would be a good idea to charge increased peak-time fares. This would be likely to reduce congestion (or distribute it across the day), preventing our trains from exceeding maximum capacity, and would give us extra profit. We will charge 1.5x the standard amount for each route for the periods from 7:00 - 8:59 and 16:00 – 17:59.

We can then superimpose the cost timetable onto our congestion data, and sum across all times and routes, in order to find our total daily profit.

SALES MODIFICATIONS

Not all of our tickets will be sold at full price. We will allow several discounts:

- Children (Travellers under 16 years of age) will be discounted 40%
- We will offer seasonal tickets, and estimate that their average overall discount will amount to one-third of the amount that would otherwise be spent
- We will offer return tickets to commuters, which will discount 20% of any full journey.

We estimate the sales profit reduction in the appendix.

Once all calculations have been completed, it is found that the final annual profit P_{total} can be written in terms of price per kilometre p_{km} as follows:

$$P_{total} = £10.46\ bn \times p_{km}$$

COST BREAKDOWN

The key costs in our project are:

- Track Construction
- Tunnelling
- Station Construction
- Trains
- Energy
- Operations & Maintenance

These will be considered in subsections.

TRACK CONSTRUCTION & TUNNELLING

Looking at the costs of lines on the Japanese Shinkansen allows us to estimate track construction and tunnelling costs. Two lines, Chūō Shinkansen and Hokuriku Shinkansen, are studied. These are EDS lines, so our construction costs should be lower. We estimate that our costs will be 85% of those calculated from the following data, since the majority of the costs are for infrastructure.

Chūō Shinkansen's construction cost was £43.2 billion at 286 km, or £151m per km, adjusted for inflation [85]. However, this route is largely comprised of deep tunnels.

Hokuriku Shinkansen's construction was £20.6 billion at 228 km (between Nagano and Kanazawa), or £90m per km, adjusted for inflation [86]. This has a simpler route than ours.

Using tunnel length data from [87], we can find that 29% of Hokuriku comprises of tunnels, while from [85] we know 90% of Chūō is tunnels. If we assume underground and overground tracks have separate but constant costs per kilometre, U and O we can solve a system of equations:

$$0.29U + 0.71O = £\,90\,m$$

$$0.90U + 0.10O = £\,151\,m$$

To find that

$$O = £\,61\,m; \quad U = £\,161\,m$$

Or, adjusted for the additional cost of EDS track vs our track,

$$O = £\,51.9\,m; \quad U = £\,136.9\,m$$

We will include the modification that single-tube tunnels are two-thirds as expensive (as they are likely to be more expensive than just one smaller-diameter TBM, but less expensive than two of them).

By applying these costs to our total route lengths (150.3km overground, 107.4km underground), this gives a total present construction cost of £7.8 billion for overground construction and £14.7 billion for underground construction.

STATIONS & TRAINS

Estimations are again difficult here. Crossrail states [84] that the "Canary Wharf Group has agreed to contribute £150m towards the costs of the new Canary Wharf Crossrail station at Canary Wharf". The Canary Wharf station is a large, overground station that can be expected to cost more than any of our stations. We can only tell that the Canary Wharf station costs in the order of hundreds of millions of pounds; therefore, we will estimate that on average, each of our overground stations will cost £300m. The underground station at City can be expected to cost more; we will allocate £600m. This leads to a total cost of £2.1 billion for stations.

Conventional six-carriage trains cost about £15m [88], or £2.5m per carriage. We can estimate that maglev carriages will cost around 3 times this amount, at an average of £7.5m per carriage. This agrees well with estimates from the Colorado AGS feasibility study [89], and our own estimations from the engineering section.

We can calculate the total required number of carriages by the following process:

- Find the maximum passenger carriages used at once per train for each route and multiplying by two (since each route requires 4 trains and has 2 directions for which carriage numbers are calculated separately).
- Add eight carriages for each route, to account for the front and back carriages.
- Add an extra 10% to account for spare carriages and carriages under maintenance.

This gives a total of 280 total carriages. Assuming the capacity of the considered carriages is the same as that of ours, this gives us a total cost of almost exactly £2 billion for trains.

OPERATIONS & MAINTENANCE (INCLUDING ENERGY)

We need to pay for operations staff, as well as for vehicle and station maintenance. We use an estimate from Colorado's Advanced Guideway System Feasibility Study, scaled linearly for track distance, to obtain a value of £80m per year in staffing.

We consume 2.367 PJ of energy a year. This is a huge amount, which will likely be negotiated to a rate lower than standard electricity costs. However, we will assume the cost of power will be the current national rate of approximately 12.2 pence per kilowatt hour, which predicts that we will spend £79.9m per year on electricity.

CONTINGENCIES

These are given below:

Title	Amount	Reason
General	10% on everything	General uncertainty in project, due to unconventional transport, unique systems such as baggage control, etc.
Track Construction	20%	Right-of-way, uncertainty in land prices/ability to use planned route
Tunnelling	20%	Uncertainty in geology and potential for unexpected tunnelling issues
Operations & Maintenance	10%	Uncertainty in component lifetimes and operating costs
Trains, Stations	30%	Significant uncertainty in costs

OVERALL

Ignoring effects of inflation, the costs of our project are given here:

One-Time Costs	Length (km)	Cost (£billions)	Contingency	New Cost (£billions)
Track	150.3	7.80	120%	9.36
Underground	107.4	14.70	120%	17.64
Trains		2.00	130%	2.60
Stations		2.10	130%	2.73
Total		32.33	110%	35.57
Annual Costs		Cost (£millions)	Contingency	Cost (£millions)
O&M		80.00	110%	88.00
Energy		79.92	100%	79.92
Total		167.92	110%	184.71

To calculate the required budget, we need to have a projected timeline for construction, and we need to know when routes will come online.

PROJECT CONSTRUCTION PROGRAMME

CONSTRUCTION TIME

TUNNELLING

Crossrail has released data on the performance of its tunnelling machines during its project [90] From this we can deduce the values in the following table.

Machine	Distance (km)	Days to complete	Distance per day (m)	Days per km
Phyllis	6.8	523	13.00	77
Ada	6.8	461	14.75	68
Elizabeth	8.3	894	9.28	108
Victoria	8.3	900	9.22	108
Sophia	2.9	386	7.51	133
Mary	2.9	360	8.06	124
Average			10.30	97

The Crossrail TBMs operated in pairs, digging the two sides of a route simultaneously. We shall operate ours in the same manner, except, of course, for the short dual-track tunnels.

OVERGROUND TRACK

The Shanghai Maglev track is 30.5km and took about 2 years and 6 months to build [91, 92]. It is entirely overground, implying the construction time per kilometre of overground track is 0.082 years, or approximately a month.

TRACK SEGMENT CONSTRUCTION TIMES

The track is split further into segments for the purposes of this section. Tunnels are labelled as α, β, γ, δ, ϵ, ζ and η. Intersections are labelled as in the Planning & Environment section as I, J and U. The following table contains calculated times and costs for each route segment.

Segment	Length (km)	Time Taken - Years	Time Taken - Months	Cost (millions)
G-U	20.3	1.66	20	£1,391.16
α	**30.2**	**8.03**	**96**	**£5,457.38**
β	**24.5**	**6.51**	**78**	**£4,427.35**
H-CH	11.8	0.97	12	£808.65
γ	**14.0**	**3.72**	**45**	**£2,529.91**
δ	**26.0**	**6.91**	**83**	**£4,698.41**
CH- ζ	9.8	0.80	10	£671.59
γ-J	9.3	0.76	9	£637.33
δ- ϵ	6.0	0.49	6	£411.18
ζ	**2.7**	**0.72**	**9**	**£487.91**
ϵ	**1.8**	**0.48**	**6**	**£325.27**
ζ-I	10.7	0.88	11	£733.27
ϵ-I	6.1	0.50	6	£418.03
I-η	23.6	1.94	23	£1,617.31
η	**8.2**	**0.67**	**8**	**£1,481.81**
η-J	8.2	0.67	8	£561.95
J-S	25.3	2.07	25	£1,733.81
L-I	19.2	1.57	19	£1,315.78

Emboldened are the tunnels, which have a different construction time per kilometre. The long tunnels will take a particularly long time to finish. The process can be accelerated by using TBMs from the γ and η tunnels once they are completed.

STATIONS

A generous estimate allows 12 months for the construction of each overground station and 18 months for the underground station at City.

PROJECT TIMELINE

With the information from the previous section in mind, an overall timeline for the project can be derived.

The following two pictures present alternate project timelines. In the first, there are two TBMs for each of the tunnels, and this means that the project takes a very long time to be completed – at minimum 10 years – and towards the end of the project, not much parallelisation of construction is possible. The second requires 4 TBMs for each of the longest three tunnels, increasing cost; however, the benefit of operation beginning sooner and the earlier profits that result outweighs this. Therefore, we will use the second picture as our schedule.

In both displayed timelines, temporary, diverted lines are brought online as soon as possible to connect airports between which a direct track is yet to be built.

Year	Quarter	Events						
1	1	CH-H Track	**CH Station**	**H Station**	Build TBMs	Test Bores		
	2							
	3							
	4							
2	1	CH-ζ Track	L-I Track	**L Station**				Commence tests on CH-H
	2							
	3							
	4	ζ-I Track						
3	1			**C Station**	δ Track & Tunnel	α Track & Tunnel	β Track & Tunnel	Open H-CH
	2							
	3		I-η Track					Open L-CH, L-H
	4	ζ Track & Tunnel						
4	1							
	2							
	3	η Track & Tunnel		ϵ Track & Tunnel				
	4							
5	1			J-S Track				
	2							
	3		η-J Track					
	4							
6	1							
	2		**S Station**					
	3							
	4	γ Track & Tunnel						
7	1			G-U Track				
	2		γ-J Track					Open S-CH, S-L, S-H
	3							
	4							
8	1		**G Station**					
	2							
	3							
	4							
9	1							
	2							
	3							Open G-C
	4							
10	1							Open C-H, C-CH, C-L & temporary
	2							C-S, G-S, G-L, G-CH, G-H via I
	3							Open C-S, G-S
	4							
11	1							Open G-H, G-CH, G-L

Year	Quarter	Events						
1	1	CH-H Track	**CH Station**	**H Station**	Build TBMs	Test Bores		
	2							
	3							
	4							
2	1	CH-ζ Track	L-I Track	**L Station**				Commence tests on CH-H
	2							
	3							
	4	ζ-I Track						
3	1			**C Station**	**G Station**	α Track & Tunnel	G-U Track	Open H-CH
	2							
	3		I-η Track					Open L-CH, L-H
	4	ζ Track & Tunnel						
4	1				δ Track & Tunnel			
	2							
	3	γ Track & Tunnel		ϵ Track & Tunnel				
	4						β Track & Tunnel	
5	1			J-S Track				
	2							
	3		η Track & Tunnel					
	4							
6	1							
	2							
	3							
	4							
7	1			η-J Track		**S Station**		Open G-H, G-CH, G-L, temp. G-S
	2							
	3				γ-J Track			
	4		ϵ-I Track	δ-ϵ Track				
8	1							Open S-CH, S-L, S-H, G-C
	2							Open C-H, C-CH, C-L, C-S, G-S

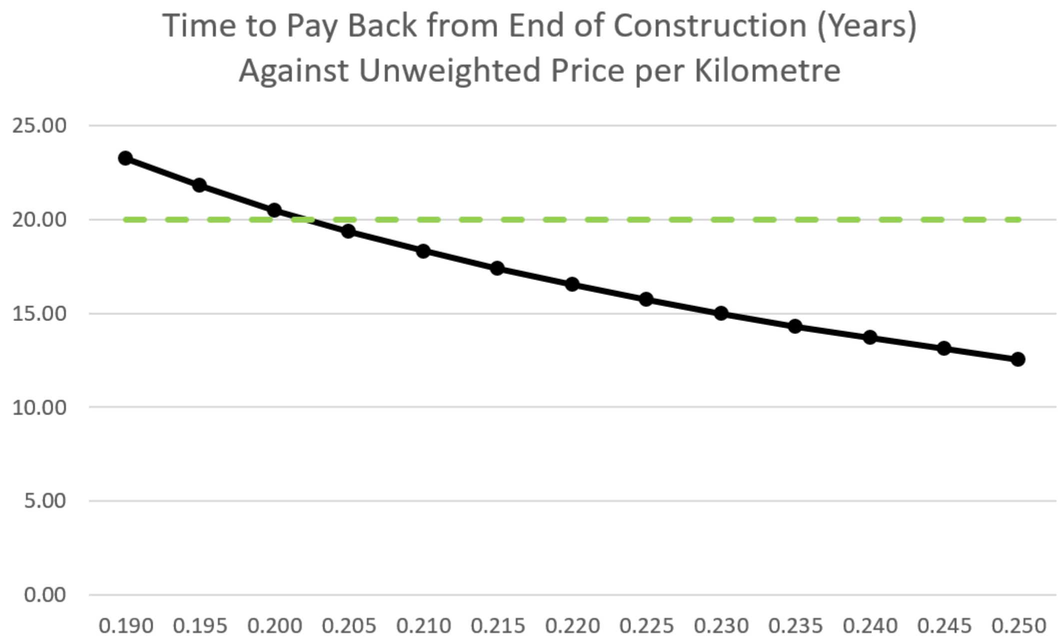
FINANCIAL PREDICTIONS

To calculate the loan payback time, we needed to do several things:

1. Find the cost and profit every year from the start of the project
2. Find the running total of cost and profit, accounting for the devaluing of loaned money due to inflation
3. Sum these values to find the cash balance each year at present value
4. Find the minimum value over time of the cash balance
5. Use this value for our total required loan at present value
6. Calculate the balance over time again, starting with the debt of the loan and applying a discount factor of the 5% interest rate to the sales profits
7. Estimate the time at which this turns positive, by finding the first positive value and linearly interpolating with the previous value

The process is shown in more detail in {15}. The loan required turns out to be **£26.725 billion**.

Varying the price per kilometre allows us to find the payback time against ticket costs:



We attain the goal of paying back the loan in 20 years with a value of £0.205 for our unweighted price per kilometre. We need to try to minimise costs for passengers, however, we should also try to minimise payback time, especially in case of unexpected issues which incur extra costs or slow system construction. As a result, we will choose an unweighted value of £0.225 per kilometre. This gives the following ticket costs, which will rise with inflation:

Route	Time (hours)																							
	0.00	1.00	2.00	3.00	4.00	5.00	6.00	7.00	8.00	9.00	10.00	11.00	12.00	13.00	14.00	15.00	16.00	17.00	18.00	19.00	20.00	21.00	22.00	23.00
G-S	£23.20	£ -	£ -	£23.20	£23.20	£23.20	£23.20	£34.80	£34.80	£23.20	£23.20	£23.20	£23.20	£23.20	£23.20	£23.20	£34.80	£34.80	£23.20	£23.20	£23.20	£23.20	£23.20	£23.20
G-C	£14.60	£ -	£ -	£14.60	£14.60	£14.60	£14.60	£22.00	£22.00	£14.60	£14.60	£14.60	£14.60	£14.60	£14.60	£14.60	£22.00	£22.00	£14.60	£14.60	£14.60	£14.60	£14.60	£14.60
G-H	£15.50	£ -	£ -	£15.50	£15.50	£15.50	£15.50	£23.20	£23.20	£15.50	£15.50	£15.50	£15.50	£15.50	£15.50	£15.50	£23.20	£23.20	£15.50	£15.50	£15.50	£15.50	£15.50	£15.50
G-L	£25.10	£ -	£ -	£25.10	£25.10	£25.10	£25.10	£37.60	£37.60	£25.10	£25.10	£25.10	£25.10	£25.10	£25.10	£25.10	£37.60	£37.60	£25.10	£25.10	£25.10	£25.10	£25.10	£25.10
G-CH	£18.00	£ -	£ -	£18.00	£18.00	£18.00	£18.00	£27.00	£27.00	£18.00	£18.00	£18.00	£18.00	£18.00	£18.00	£18.00	£27.00	£27.00	£18.00	£18.00	£18.00	£18.00	£18.00	£18.00
S-G	£23.20	£ -	£ -	£23.20	£23.20	£23.20	£23.20	£34.80	£34.80	£23.20	£23.20	£23.20	£23.20	£23.20	£23.20	£23.20	£34.80	£34.80	£23.20	£23.20	£23.20	£23.20	£23.20	£23.20
S-C	£15.50	£ -	£ -	£15.50	£15.50	£15.50	£15.50	£23.20	£23.20	£15.50	£15.50	£15.50	£15.50	£15.50	£15.50	£15.50	£23.20	£23.20	£15.50	£15.50	£15.50	£15.50	£15.50	£15.50
S-H	£25.10	£ -	£ -	£25.10	£25.10	£25.10	£25.10	£37.60	£37.60	£25.10	£25.10	£25.10	£25.10	£25.10	£25.10	£25.10	£37.60	£37.60	£25.10	£25.10	£25.10	£25.10	£25.10	£25.10
S-L	£21.40	£ -	£ -	£21.40	£21.40	£21.40	£21.40	£32.10	£32.10	£21.40	£21.40	£21.40	£21.40	£21.40	£21.40	£21.40	£32.10	£32.10	£21.40	£21.40	£21.40	£21.40	£21.40	£21.40
S-CH	£22.30	£ -	£ -	£22.30	£22.30	£22.30	£22.30	£33.40	£33.40	£22.30	£22.30	£22.30	£22.30	£22.30	£22.30	£22.30	£33.40	£33.40	£22.30	£22.30	£22.30	£22.30	£22.30	£22.30
C-G	£14.60	£ -	£ -	£14.60	£14.60	£14.60	£14.60	£22.00	£22.00	£14.60	£14.60	£14.60	£14.60	£14.60	£14.60	£14.60	£22.00	£22.00	£14.60	£14.60	£14.60	£14.60	£14.60	£14.60
C-S	£15.50	£ -	£ -	£15.50	£15.50	£15.50	£15.50	£23.20	£23.20	£15.50	£15.50	£15.50	£15.50	£15.50	£15.50	£15.50	£23.20	£23.20	£15.50	£15.50	£15.50	£15.50	£15.50	£15.50
C-H	£19.60	£ -	£ -	£19.60	£19.60	£19.60	£19.60	£29.50	£29.50	£19.60	£19.60	£19.60	£19.60	£19.60	£19.60	£19.60	£29.50	£29.50	£19.60	£19.60	£19.60	£19.60	£19.60	£19.60
C-L	£17.20	£ -	£ -	£17.20	£17.20	£17.20	£17.20	£25.70	£25.70	£17.20	£17.20	£17.20	£17.20	£17.20	£17.20	£17.20	£25.70	£25.70	£17.20	£17.20	£17.20	£17.20	£17.20	£17.20
C-CH	£18.00	£ -	£ -	£18.00	£18.00	£18.00	£18.00	£27.00	£27.00	£18.00	£18.00	£18.00	£18.00	£18.00	£18.00	£18.00	£27.00	£27.00	£18.00	£18.00	£18.00	£18.00	£18.00	£18.00
H-G	£15.50	£ -	£ -	£15.50	£15.50	£15.50	£15.50	£23.20	£23.20	£15.50	£15.50	£15.50	£15.50	£15.50	£15.50	£15.50	£23.20	£23.20	£15.50	£15.50	£15.50	£15.50	£15.50	£15.50
H-S	£25.10	£ -	£ -	£25.10	£25.10	£25.10	£25.10	£37.60	£37.60	£25.10	£25.10	£25.10	£25.10	£25.10	£25.10	£25.10	£37.60	£37.60	£25.10	£25.10	£25.10	£25.10	£25.10	£25.10
H-C	£19.60	£ -	£ -	£19.60	£19.60	£19.60	£19.60	£29.50	£29.50	£19.60	£19.60	£19.60	£19.60	£19.60	£19.60	£19.60	£29.50	£29.50	£19.60	£19.60	£19.60	£19.60	£19.60	£19.60
H-L	£16.30	£ -	£ -	£16.30	£16.30	£16.30	£16.30	£24.50	£24.50	£16.30	£16.30	£16.30	£16.30	£16.30	£16.30	£16.30	£24.50	£24.50	£16.30	£16.30	£16.30	£16.30	£16.30	£16.30
H-CH	£ 5.50	£ -	£ -	£ 5.50	£ 5.50	£ 5.50	£ 5.50	£ 8.20	£ 8.20	£ 5.50	£ 5.50	£ 5.50	£ 5.50	£ 5.50	£ 5.50	£ 5.50	£ 8.20	£ 8.20	£ 5.50	£ 5.50	£ 5.50	£ 5.50	£ 5.50	£ 5.50
L-G	£25.10	£ -	£ -	£25.10	£25.10	£25.10	£25.10	£37.60	£37.60	£25.10	£25.10	£25.10	£25.10	£25.10	£25.10	£25.10	£37.60	£37.60	£25.10	£25.10	£25.10	£25.10	£25.10	£25.10
L-S	£21.40	£ -	£ -	£21.40	£21.40	£21.40	£21.40	£32.10	£32.10	£21.40	£21.40	£21.40	£21.40	£21.40	£21.40	£21.40	£32.10	£32.10	£21.40	£21.40	£21.40	£21.40	£21.40	£21.40
L-C	£17.20	£ -	£ -	£17.20	£17.20	£17.20	£17.20	£25.70	£25.70	£17.20	£17.20	£17.20	£17.20	£17.20	£17.20	£17.20	£25.70	£25.70	£17.20	£17.20	£17.20	£17.20	£17.20	£17.20
L-H	£16.30	£ -	£ -	£16.30	£16.30	£16.30	£16.30	£24.50	£24.50	£16.30	£16.30	£16.30	£16.30	£16.30	£16.30	£16.30	£24.50	£24.50	£16.30	£16.30	£16.30	£16.30	£16.30	£16.30
L-CH	£13.70	£ -	£ -	£13.70	£13.70	£13.70	£13.70	£20.60	£20.60	£13.70	£13.70	£13.70	£13.70	£13.70	£13.70	£13.70	£20.60	£20.60	£13.70	£13.70	£13.70	£13.70	£13.70	£13.70
CH-G	£18.00	£ -	£ -	£18.00	£18.00	£18.00	£18.00	£27.00	£27.00	£18.00	£18.00	£18.00	£18.00	£18.00	£18.00	£18.00	£27.00	£27.00	£18.00	£18.00	£18.00	£18.00	£18.00	£18.00
CH-S	£22.30	£ -	£ -	£22.30	£22.30	£22.30	£22.30	£33.40	£33.40	£22.30	£22.30	£22.30	£22.30	£22.30	£22.30	£22.30	£33.40	£33.40	£22.30	£22.30	£22.30	£22.30	£22.30	£22.30
CH-C	£18.00	£ -	£ -	£18.00	£18.00	£18.00	£18.00	£27.00	£27.00	£18.00	£18.00	£18.00	£18.00	£18.00	£18.00	£18.00	£27.00	£27.00	£18.00	£18.00	£18.00	£18.00	£18.00	£18.00
CH-H	£ 5.50	£ -	£ -	£ 5.50	£ 5.50	£ 5.50	£ 5.50	£ 8.20	£ 8.20	£ 5.50	£ 5.50	£ 5.50	£ 5.50	£ 5.50	£ 5.50	£ 5.50	£ 8.20	£ 8.20	£ 5.50	£ 5.50	£ 5.50	£ 5.50	£ 5.50	£ 5.50
CH-L	£13.70	£ -	£ -	£13.70	£13.70	£13.70	£13.70	£20.60	£20.60	£13.70	£13.70	£13.70	£13.70	£13.70	£13.70	£13.70	£20.60	£20.60	£13.70	£13.70	£13.70	£13.70	£13.70	£13.70

Ticket prices, by route, by hour

With these prices, we will be able to pay back our load **15 years, 8 months and 16 days** after construction has finished, or **24 years, 2 months and 16 days** after construction has begun.


:20

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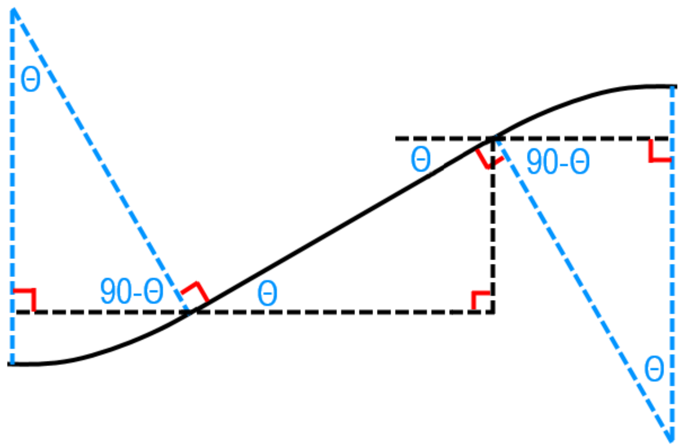
CALCULATIONS

1. ALTITUDE CHANGES

The bridges in our system work by raising the track to the correct height, remaining at that level whilst crossing over the obstacle (most commonly a road), then returning to 2 m above ground level after passing the obstacle. A change of height of track is necessary when it must go over roads or underground, however, to minimise the use of excessive high tracks and minimise underground length, the track should remain at ground level for as long as possible before either going up to bridge level or down underground.

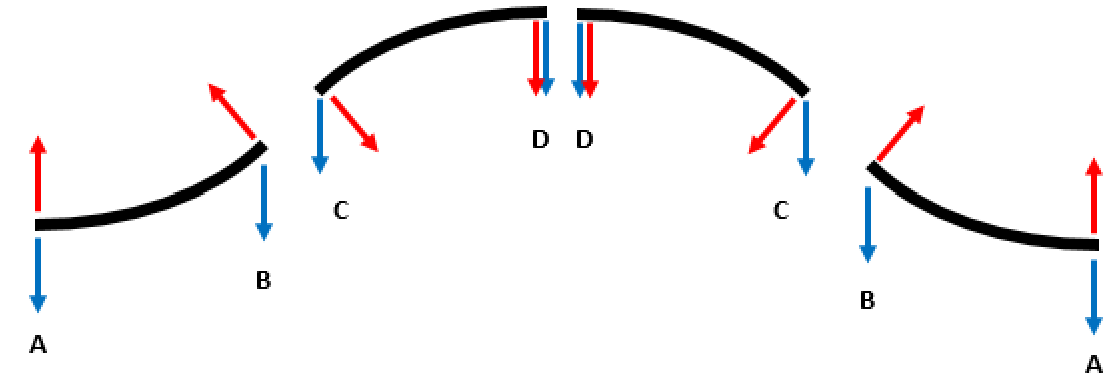
However, the rate of change of gradient is restricted by the maximum G-Force that can be exerted on passengers horizontally. As a maximum acceleration, we impose the value of 0.72 ms^{-2}, because it is comfortable to all passengers, including the elderly. The same value is used for the Shinkansen trains [93]. We can work out the minimum distance over which the track can increase 10% in gradient.

A 10% increase in gradient from the horizontal corresponds to an angle $\tan^{-1}\left(\frac{1}{10}\right)$ or approximately 5.71 degrees from the horizontal. In order to attain this gradient, the track needs to curve upwards on an arc which sweeps the same angle.



The acceleration exerted on the passengers as the train passes the curved sections can be calculated by working out the horizontal and vertical components of the transport accelerating towards the centre of the circle as it changes in gradient, calculated by $\frac{v^2}{r}$. The maximum speed of the system at any point is 100 ms^{-1}, so the acceleration towards the centre of the circle is equal to $\frac{10000}{r}ms^{-2}$.

The maximum acceleration exerted on passengers will be at one of four points, labelled A, B, C and D, each representing a point where horizontal or vertical components of the acceleration are at their largest:



Point	Acceleration $/ms^{-2}$ (up & right taken as positive)		
	Vertical (gravity)	Vertical (centripetal)	Horizontal (centripetal)
A	-9.81	$10000/r$	0.00
B	-9.81	$-10000\cos(5.71°)/r$	$10000\sin(5.71°)/r$
C	-9.81	$10000\cos(5.71°)/r$	$-10000\sin(5.71°)/r$
D	-9.81	$10000/r$	0.00

Key	
Colour	Meaning
Red	Centripetal acceleration
Blue	Acceleration due to gravity
Black	Track

As we know that the horizontal component is most likely the limiting factor, we can work out the minimum radius of curvature with the equation

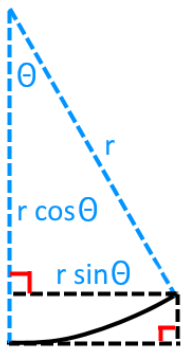
$$\frac{v^2\sin(5.71°)}{r} = 0.72$$

Or, rearranging

$$r = \frac{v^2\sin(5.71°)}{0.72} \approx 1400\ m$$

We use a conservative value of 100 ms^{-1} for v, and round the result up to

By placing a value of 1400m into the horizontal components, the most acceleration the passengers will feel is a value of 16.95 ms^{-2} (to 2 d.p.at point) at point A. This is a G-force of just over 1.73g, which is well within the comfort limit for vertical acceleration. Thus, the minimum radius of the circle which the track follows when changing gradient is 1400m.



In order for a train to curve up to a gradient of 10% at radius r_0 it must travel a distance of B metres, going up an elevation of A metres, as shown in this diagram. We can see geometrically that

$$A = r - r\cos 5.71°$$

$$B = r\sin 5.71°$$

The same will hold for the other curved segments.

Adding the contributions from the different segments, we find that the total height and width h and w of height-transition section

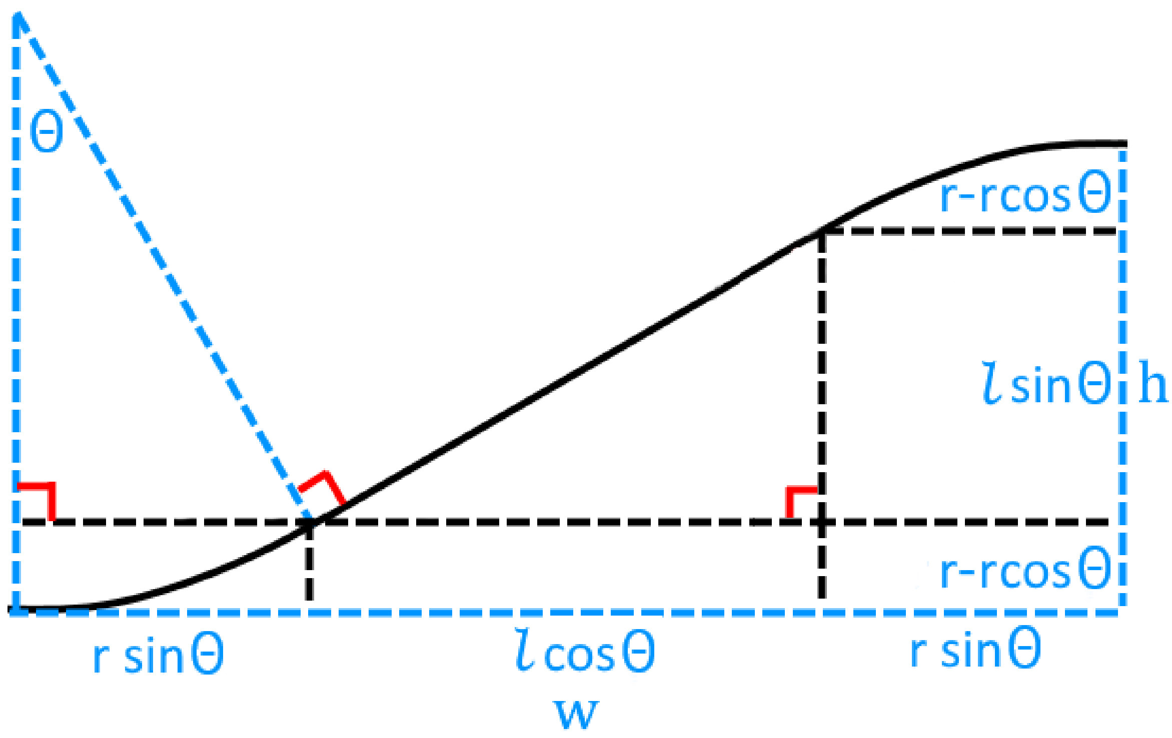
$$h = 2r(1 - \cos 5.71°) + l\sin 5.71°$$

$$w = 2r\sin 5.71° + l\cos 5.71°$$

We can substitute in the values of the trig functions, and eliminate the middle section length to find the required width of the height-transition section for a given height change:

$$l \approx 10.05h - 0.010r$$

$$\Rightarrow w \approx 10.00h + 0.189r$$



By this equation, in order to pass from the above ground tracks (2m above ground) to the underground tunnels, at around 40m below ground, we will need approximately 665m of height-transitioning track.

In order to reach the required bridge height of 12m (10m above normal height), which allows for clearance over even the 5.1m high motorway bridges that are present in some places, approximately 365m of height-transitioning track must be used.

2. COST OF HIGH SPEED RAIL

From estimations in {X}, we can multiply the total carriages for each route by each route's distance to obtain a total travelled distance per day by all trains of 399473 km.

The total distance travelled per year is then $365 \times 399473 \approx 146 \times 10^9 m$.

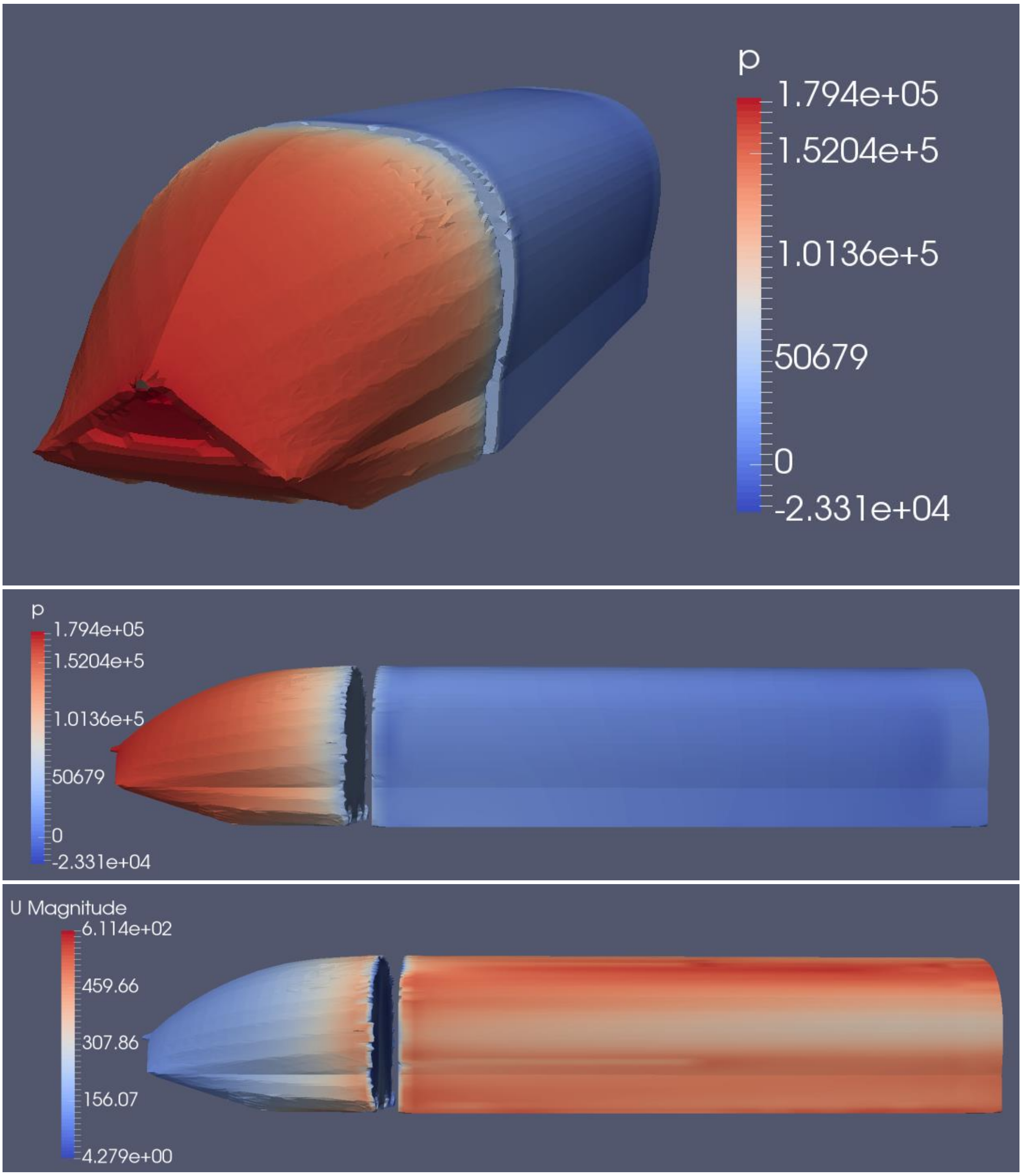
Converting to miles, and using a cost of $20 per trainset-mile [94] at an exchange rate of 0.82 GBP per USD, we obtain a total maintenance cost per year of £1.49 billion.

This is a huge amount of money, and illustrates why a maglev system would be an effective solution for this problem.

3. CFD ANALYSIS

CFD (computational fluid dynamics) is used in many industries to predict surface pressures on an object due to airflow as well as to produce estimates for the drag force that will be experienced. In our case, simulations were performed using SimFlow. This is a freeware application which functions as a GUI for OpenFOAM (an open source collection of tools for modelling the flow of fluids in different scenarios). A model for the train was created through Google Sketchup and then converted to an STL which could be imported.

Within SimFlow, the model then had to be meshed which converts it into a format usable in the simulations. Due to limitations imposed by the free version of this software, the mesh could only be created with a maximum of 100,000 nodes. For the simulation parameters, we used the RANS turbulent flow model which can accurately predict the mean turbulence of the air. The train was simulated as being in an enclosed space to account for greater surface pressures when travelling through tunnels underground. For the air flow, a speed of $100\ ms^{-1}$ was used. It is possible that, when travelling into a head wind, the train will experience a velocity greater than this, however, large safety margins will be used during the structural design which will account for this. All walls around the train were configured to move at the same velocity as the airflow to simulate the interior of a tunnel. The results are shown below:



From top to bottom:

1 – View of the surface pressure (in Pa) experienced at the front of the train. The indented section mimics the opening for air to enter the train

2 – Surface pressure (in Pa) along the length of the train. As can be seen, pressure is concentrated at the head of the train.

3 – Air velocity (in ms^{-1}) along the length of the train. This demonstrates why skin friction will be the primary cause for drag on the train even after taking into consideration the boundary layer of turbulence that will occur.

4. TRAIN SKIN PRESSURE

PRESSURE ON SKIN (FROM CFD):	
AT TRAIN HEAD	180 kPa
ALONG CARRIAGE	25 kPa
SKIN THICKNESS	1.5 mm
SHEAR STRENGTH	150 MPa [95]
SHEAR MODULUS	26.9 GPa

The skin is thicker than some aircraft, but still relatively thin to reduce weight.

We don't want the skin to deform more than 0.4mm so using the shear modulus we can calculate the pressure required for this to happen:

$$G = \frac{\tau}{\Delta x / l}$$

$$26.9 \times 10^9 = \frac{\tau}{0.4 \times 10^{-3} / (x \times 0.5)}$$

Solving for tau (shear stress):

$$\tau = \frac{5.38}{x} MPa$$

This is less than the shear strength so the skin will not break before this point.

Let x be the length between successive bulkheads, and let L be the theoretical width of the section of skin being measured

Force from air:

$$F = PA$$

$$F = PxL$$

The reaction force is split between both bulkheads. Therefore, shear force is halved.

$$F = 0.5PxL$$

Shear stress can be calculated as:

$$\tau = \frac{F}{A}$$

$$\frac{5.38 * 10^6}{x} = \frac{0.5PxL}{L \times 1.5 \times 10^{-3}}$$

Solving for maximum value of x:

$$x = \sqrt{\frac{5380}{P}}$$

At the head of the train, this gives 0.299m. Along the carriages, this gives 0.803m.

5. REQUIREMENT FOR HEATING

Assumed U-value for train wall	0.2 $Wm^{-2}K$

This value is slightly greater than that of a foam insulated brick wall [96]

The train must be able to maintain a temperature difference of 25°C and has a surface area of approximately 186m2 (modelled as a 14x3x3m cuboid)

We can calculate the power requirement for carriage heating using:

$$P = uA\Delta T = 0.2 \times 186 \times 25 = 930\ W$$

We shall use 1200W to add a margin for error and account for inefficiencies in the heating solution.

6. MAGNETICS SIMULATIONS

FEMM is an open source program for analysing magnetostatic systems. It was used to produce estimates of levitation and propulsion forces in our maglevs. Propulsion estimations were simple: a Halbach array was created using neodymium magnets and placed over a series of track coils. The coils in front were powered to attract the Halbach array whereas the coils behind were powered to repel. FEMM makes this process simple since circuits can be created to represent coils and the current in each circuit can be set.

For the levitation system, simulations were more complicated because it relies on the movement of the train to produce the magnetic field in the track. Since FEMM allows the use of Lua scripts, we created a script to slowly move the Halbach array over the track coils and calculate the potential difference in the coils according to Faraday's law:

$$E = -N\frac{\Delta\phi}{\Delta t}$$

This could then be used to calculate the current according to:

$$V = IR + L\frac{dI}{dt}$$

Which models the change in current for a series resistor-inductor circuit where L is the inductance of the coil. This in turn was calculated according to:

$$L = \frac{N^2\mu A}{l}$$

The current calculated was then used in a second pass of the simulation which was used to calculate the repulsive levitation force by integrating the magnetic stress tensor over the area of the Halbach array. This could then be used to determine the density of the coils required to levitate the train. A video of the propulsion simulation results is included in the linked YouTube video [97]. The channel will also be releasing several more animations in the near future.

The simulation showed that the train will lift off the ground at a speed of 20 km/h.

7. BATTERY REQUIREMENTS

We know that

$$Energy(Wh) = Power(W) \times Time(h)$$

$$\Rightarrow E = 3600 \times 3 = 10.8\ kWh$$

Giving a requirement of 10,800Wh

$$\frac{10800}{160} = 67.5kg$$

Therefore, we shall equip each carriage with 70kg worth of lithium ion batteries.

8. TRAIN KINEMATICS

Let us first model the trajectory of the train in one direction, with a constant driving force. The equation of motion is

$$ma = D - R$$

Where D is the driving force, R is the resistance to motion, a is the acceleration and m is the mass.

Resistance for trains has been modelled for over 80 years by the quadratic Davis equation:

$$R = AW + Bv + Cv^2$$

Where R is the rail vehicle resistance, v is the velocity of the vehicle, and A, B and C are regression coefficients obtained by fitting test data to the Davis equation, all in SI units. There are more modern, updated versions that are slightly more complicated. However, in every equation, almost all terms depend on frictional losses. Obviously, none of these terms will have an effect for our vehicle, as it has no moving parts and makes no contact with the tracks. The only term left over is the air resistance. [98, 99]

Air resistance can be modelled by the turbulent drag equation:

$$F = kv^2 = \frac{1}{2}\rho A C_d v^2$$

Where ρ is ambient air density, A is the cross-sectional area of the train in the direction of motion, and C_d is the drag coefficient, a dimensionless number that depends on the train's geometry. The first two of these numbers are easily determined: Average atmospheric air density is $1.2754\ kg/m^3$, and the frontal surface area of the train is approximately $12\ m^2$.

The drag coefficient is more difficult. There are two main types of drag – frontal, where air hits the front of a moving object and high pressure builds up when it cannot escape, and skin friction, which as the name implies arises from friction with the surface of the object as air moves past it. Normally, frontal drag is more significant, but trains can be long enough for skin friction to play an important part. Numerically, the drag coefficient is approximately dependent on the logarithm of the Reynolds number, which depends proportionally on both the length of the train and the velocity.

We were unable to find a well-cited and detailed source on the drag coefficient of a train, and papers on the calculation of such drag coefficients were beyond our level of understanding. As such we estimate the drag coefficient to be 0.26, due to streamlining of the front of the train, and considering the coefficient of the Transrapid [8]

Using the turbulent drag equation, the equation of motion can also be written as

$$m\dot{v} = D - kv^2$$

Or, rearranging,

$$\dot{v} = \frac{D}{m}\left(1 - \frac{k}{D}v^2\right)$$

Then letting

$$A = \frac{D}{m}$$

$$B = \sqrt{\frac{k}{D}}$$

We have

$$\dot{v} = A(1 - B^2v^2)$$

$$\frac{1}{1 - B^2v^2} \cdot \frac{dv}{dt} = A$$

So

$$\int \frac{1}{1 - B^2v^2} dv = \int A\, dt$$

We can use trigonometric substitution to solve this integral:

$$Bv = \tanh(u)$$

$$\tanh^{-1}(Bv) = u$$

$$\Rightarrow \frac{1}{1 - B^2v^2} dv = du$$

So now we have

$$\int du = \int A\, dt$$

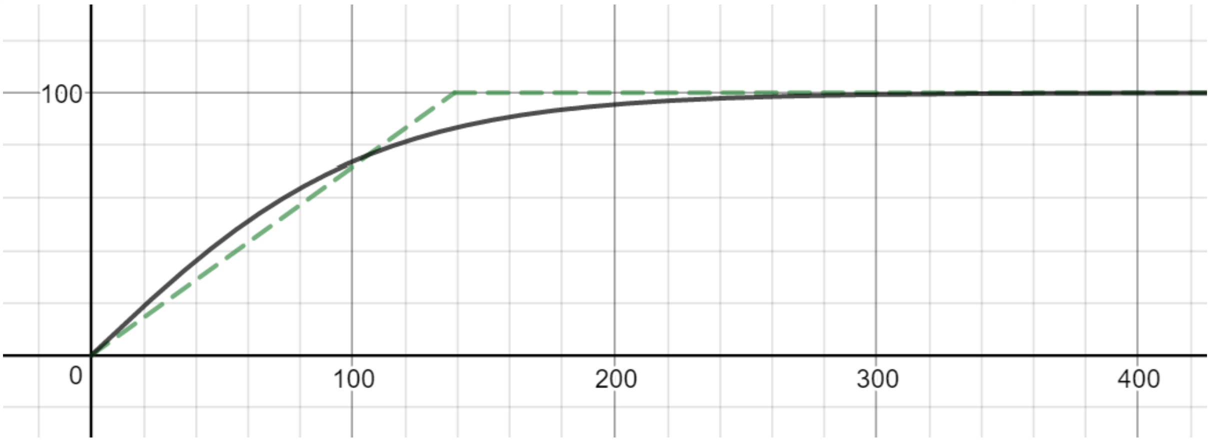
Or

$$u = At + C$$

$$\Longrightarrow \tanh^{-1}(Bv) = At + C$$

$$v(t) = \frac{\tanh(\mathrm{At + C})}{B} = \boxed{\sqrt{\frac{D}{k}} \tanh\left(\frac{D}{m}t\right)}$$

We have set C equal to zero because the velocity at time 0 should be 0. Here is a graph of this velocity over time:



Velocity (m/s) vs time (s)

It is clear that the green line, the first section of which corresponds to $v = at$, could almost exactly be followed by varying the initial driving force. This gives the equation of motion at the maximum permitted acceleration. When approaching the maximum velocity, the acceleration would reduce slightly if the maximum driving force were close in magnitude to the air resistance experienced. However, the maximum possible driving force should be greater than this anyway, to allow for (for example) combatting strong winds.

Using this constant acceleration would maximise the distance travelled by the train in a given time. Therefore, we should instead model the motion of the train more simply, using the "suvat" equations.

After a time t_1, the train approaches its destination and begins to decelerate. Similarly, we will assume this happens at a constant rate a which is just less than the maximum acceptable number of $1\ ms^{-2}$. This is possible by fine control (and eventual reversal) of the decreasing driving force as the train slows.

We have an equation for the velocity

$$v(t) = \begin{cases} at, & 0 \leq t < t_a \\ V, & t_a \leq t < T - t_a \\ -at, & T - t_a \leq t < T \end{cases}$$

Where T is the total journey time, V is the maximum velocity of the train, given by setting the force of the system equal to 0:

$$0 = D - kV^2$$

$$\Rightarrow V = \sqrt{\frac{D}{v}}$$

And t_a is the time taken to accelerate, given by $\frac{V}{a}$.

Integrating each part of this graph to find the total distance travelled, we find

$$S = V(T - 2t_a) + \frac{2at_a^2}{2} = VT - \frac{V^2}{a}$$

We can use this formula, rearranged to make a the subject, to find the required acceleration for various velocities to travel the longest route in 20 minutes:

$$a(S - VT) = -V^2$$

$$a = \frac{V^2}{(VT - S)}$$

We can also find the work done by the train in a journey. To maintain constant acceleration or deceleration, the difference between the driving force and the resistive force needs to be constant. This means that the driving force must be held at $kv^2 \pm ma$, where m is the mass of the train. Then, during acceleration and deceleration,

$$F = D - R = \pm ma$$

To find the work done during acceleration, we need to evaluate the integral

$$\int_0^{t_a} P(t) \cdot dt = \int_0^{t_a} D(t) \cdot v(t) \cdot dt = \int_0^{t_a} \big(kv(t)^3 + mav(t)\big) \cdot dt$$

$$= \int_0^{t_a} (k[at]^3 + ma[at]) \cdot dt = \frac{ka^3 t_a^4}{4} + \frac{ma^2 t_a^2}{2}$$

$$= \frac{kV^4}{4a} + \frac{1}{2}mV^2$$

The work done during deceleration is similarly

$$\int_0^{t_a} (k[V - at]^3 - ma[V - at]) \cdot dt = \int_0^{t_a} (k[V^3 - V^2 at + Va^2 t^2 - a^3 t^3] - ma[V - at]) \cdot dt$$

$$= -\frac{ka^3 t_a^4}{4} + \frac{kVa^2 t_a^3}{3} - \frac{kV^2 a t_a^2}{2} + kV^3 t_a + \frac{ma^2 t_a^2}{2} - maVt_a$$

$$= -\frac{kV^4}{4a} + \frac{kV^4}{3a} - \frac{kV^4}{2a} + \frac{kV^4}{a} + \frac{mV^2}{2} - mV^2 = \frac{7kV^4}{12a} - \frac{1}{2}mV^2$$

These two expressions separate into terms for work against air resistance and terms that cancel to give the kinetic energy change. If the only acceleration and deceleration is at the start and end of the journey, for the period when the train is travelling at maximum velocity, the work done is

$$Fd = kV^2 \cdot V(T - 2t_a) = kV^3 T - \frac{2kV^4}{a}$$

And this means that the work done throughout the journey is

$$W = \frac{kV^4}{4a} + \frac{1}{2}mV^2 + kV^3 T - \frac{2kV^4}{a} + \frac{7kV^4}{12a} - \frac{1}{2}mV^2$$

$$= kV^3 T - \frac{7kV^4}{6a}$$

A useful application of this formula is to evaluate the most efficient velocity for various travel distances. We have a formula relating journey time and distance, allowing us to find work done in terms of velocity:

$$S = VT - \frac{V^2}{a}$$

$$\Rightarrow T = \frac{S}{V} + \frac{V}{a}$$

$$\Rightarrow W = kV^3\left(\frac{S}{V} + \frac{V}{a}\right) - \frac{7kV^4}{6a}$$

$$W = kV^2\left(S - \frac{V^2}{6a}\right)$$

The energy efficiency of the Transrapid is around 85%. Our maglev system will be similar, so the total energy used by our train for a journey is given by:

$$E = kV^2\left(S - \frac{V^2}{6a}\right)$$

This equation can be plotted on a three-dimensional graph, with velocity as the x-axis, distance as the y-axis, and energy consumption on the z-axis. We add the constraint that the journey time must be less than 20 minutes of 1200 seconds, given algebraically by

$$T = \frac{S}{V} + \frac{V}{a} \leq 1200$$

$$\Longrightarrow V^2 - 1200aV + aS \leq 0$$

The following graphs result:

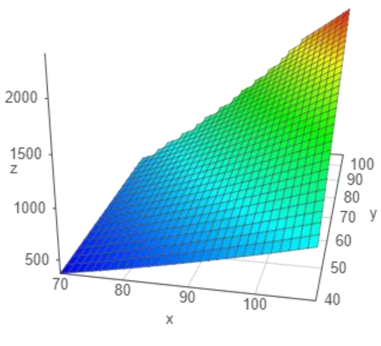


Figure 1 – $a = 1\ ms^{-2}$, view 1

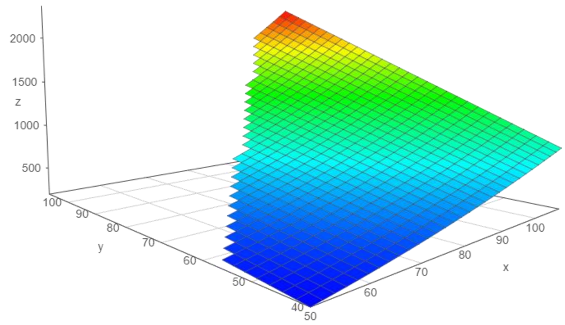


Figure 2 – $a = 1\ ms^{-2}$, view 2

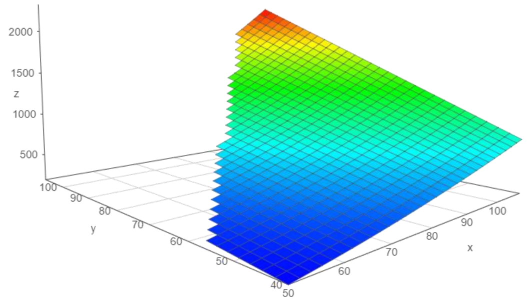


Figure 3 – $a = 0.5\ ms^{-2}$

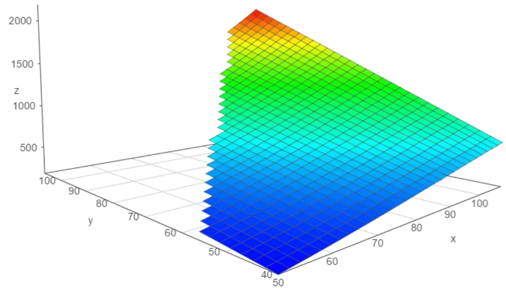


Figure 4 – $a = 0.2\ ms^{-2}$

It is clear that changing acceleration has little effect on the general form of the graph. It is also clear that the velocity should be reduced for shorter journeys in order to minimise energy usage.

The equality case, where

$$V^2 - 1200aV + aS = 0$$

Clearly minimises the energy for a given S. We can therefore solve for V to find the minimum for each journey:

$$V = 600a \pm \sqrt{360{,}000 \times a^2 - aS}$$

The case with a positive radical is unphysical, as it predicts a velocity that decreases as the distance increases.

For the routes involved in our solution, this gives the following velocities and energies for the routes when the acceleration is $0.72\ ms^{-2}$:

ROUTE	DISTANCE (km)	VELOCITY (m/s)	TRAINS /DAY {X}	ENERGY CONSUMED	
				Single Trip (MJ)	Daily (GJ)
G-S	93.4	86.5	458	1310.5	600.2
G-C	44.8	39.1	382	129.9	49.6
G-H	50.5	44.4	516	188.2	97.1
G-L	106.0	99.9	476	1976.8	940.9
G-CH	63.6	56.7	378	386.5	146.1
S-C	48.6	42.6	299	167.1	50.0
S-H	103.6	97.3	492	1834.2	902.4
S-L	86.5	79.4	354	1023.9	362.5
S-CH	90.5	83.5	306	1183.8	362.3
C-H	76.2	69.0	458	683.5	313.0
C-L	59.1	52.4	302	307.1	92.8
C-CH	63.1	56.2	244	377.0	92.0
H-L	55.5	49.0	494	252.4	124.7
H-CH	11.8	9.9	422	2.2	0.9
L-CH	42.4	36.9	346	109.6	37.9
SUM/MAX	**996.9**	**99.9**	**5927**	**9932.8**	**4172.4**

These ideal values do not reflect the true ones exactly. This is because trains on the same segment of track must travel at the same speeds, increasing the velocity for some tracks. We will add 20% to the total energy to account for this.

Train operation will constitute the vast majority of power use. To account for other power requirements such as station and train lighting, we will add a further 10%. Finally, we can assume our system to be about as efficient as rapid, with 85% of energy converted to useful output. We find the following values:

ENERGY CONSUMED:	
PER DAY (GJ)	6479.5
PER YEAR (TJ)	2366.7

STOPPING DISTANCES

The distance between two consecutive trains should be equal to the stopping distance of a train. Trains can always stop in a controlled manner, even if a power supply is lost, because of their inductive power regeneration function. The stopping distance is then given by using the maximum acceptable deceleration with the constant-velocity "suvat" equations:

$$v^2 = u^2 + 2as$$

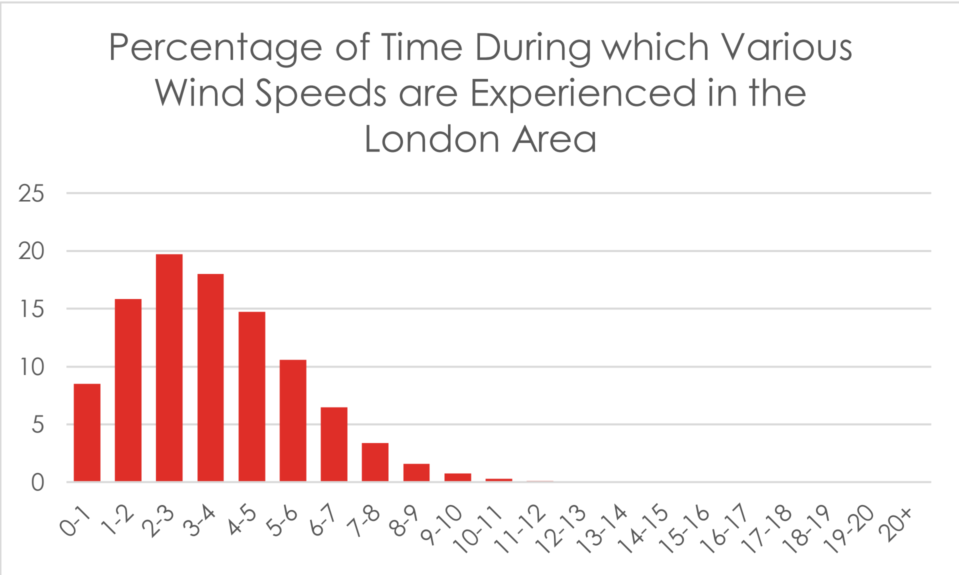
$$s = \frac{u^2}{2a} = 6.4\ km$$

An allowance should be made for 5 seconds travelling at full velocity due to communication protocols discussed in the engineering section. In an emergency, a higher acceleration than our normal comfortable value can be used. Horizontally, 2 ms^{-2} is still perfectly safe for passengers, especially for short periods [100]. Adding the 5 second allowance, and substituting our maximum possible velocity (which will give us the worst-case scenario relative to the train speeds due to the v^2 term) with this acceleration gives us

$$s = 5v + \frac{v^2}{2a} = 2.784\ km$$

This distance between consecutive trains is easily feasible. The busiest section of our track runs from the intersection point to the Central Hub, with 7 trains every 5 minutes. Staggering these maximally gives a distance between consecutive trains of 4.114 km.

WIND SPEEDS [101]



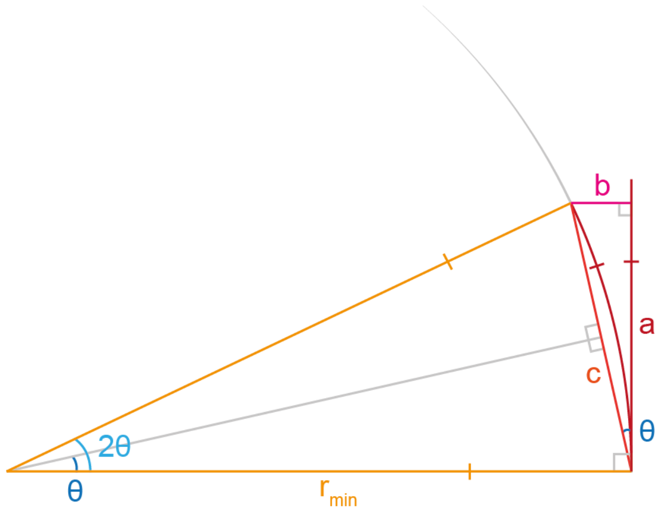
If the trains can combat 12 km/h or 3.33 ms^{-1} winds at full speed, they will be able to operate at full speed 99.3% of the time. Wind speeds greater than 15 kmh^{-1} or 4.17 ms^{-1} occur less than 0.005% of the time. Withstanding wind speeds of 5 ms^{-1} should therefore mean wind is almost never a cause of delays.

Doing this is equivalent to experiencing drag, calculated with the standard equation, but with the train's velocity increased by 5 ms^{-1}. The driving force the engine must be able to exert is then equal to the drag experienced:

$$D_{max} = k \cdot (v_{max} + 5)^2 \approx 21.9\ kN$$

9. SWITCHING SECTION DIMENSIONS

The length of the arc is the same as the length of the track when it is in straight position (assuming the bending of the track does not change its length). The following geometric representation of the track in its straight and turnout positions can be constructed:



Where a is the length of the track in its straight position, b is the maximum sideways displacement attained, r_{min} is the minimum turning radius of the train, c is the straight line connecting the two ends of the arc, and θ is the angle between a and c in radians. The minimum possible value of b is the width of the track.

By geometric arguments, 2θ is the angle between the chords of the arc. Then, the length of the arc a can be found in terms of r and θ:

$$a = 2\theta r_{min}$$

Since b and r are the variables that are constrained, it would be helpful to find θ in terms of b. We can find the following relationships:

$$\sin\theta = \frac{c}{2r_{min}}$$

$$c = \frac{b}{\sin\theta}$$

Combining the two equations and solving for θ, we find

$$\sin\theta = \sqrt{\frac{b}{2r_{min}}}$$

For the range of values that we are interested in, θ will be very small, because the turning radius is very large. Therefore, we can use to a high degree of accuracy the approximation that

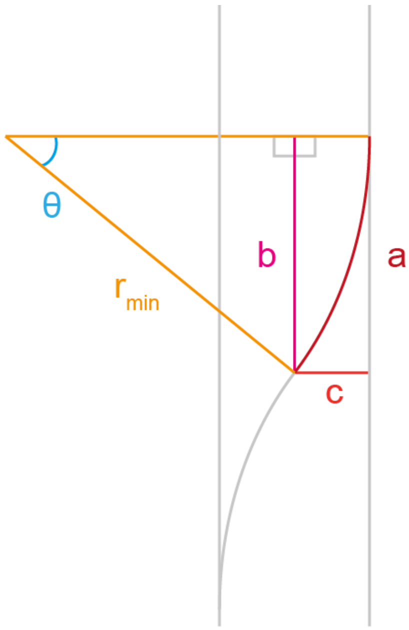
$$\sin\theta \approx \theta$$

Applying this, and substituting into the equation for a, we obtain an expression for a in terms of b and r:

$$a = \sqrt{2br_{min}} = 211.6\ m$$

At low speeds, such as when a train is inside a station and traveling on wheels, another switching solution can be implemented. This relies on the contact between the wheels and the rail, and is similar to that of a conventional rail switch. The switching distance is then allowed be relatively small.

This solution is different from a conventional track switch in that specific sections of the guide rails need to be lowered by hydraulic pistons, in order to make space for the levitation system of the vehicle, which hangs below the wheels.



Where a is the length of half of the track switch, b is the component of a parallel to the direction of the track, and c is half of the distance between the two tracks. r_{min} is now the minimum turning radius at low speed, and θ is the angle between the two chords of arc a in radians. In this situation, the minimum turning radius and the distance between the tracks are the constraints, and their relationships with the other measurements can be found:

$$a = \theta r_{min}$$

$$\sin\theta = \frac{b}{r_{min}}$$

$$r_{min}^2 = b^2 + (r_{min} - c)^2$$

$$\Rightarrow b = \sqrt{2r_{min}c - c^2}$$

$$\Rightarrow \theta = \sin^{-1}\left(\frac{\sqrt{2r_{min}c - c^2}}{r_{min}}\right)$$

$$\Rightarrow a = \sin^{-1}\left(\frac{\sqrt{2r_{min}c - c^2}}{r_{min}}\right) r_{min}$$

At 10 ms^{-1}, which trains should be below at these sections, $r_{min} = v^2/a_{max} = 139\ m$, so $a = 23.6\ m$.

10. PRESSURE WAVES

1. **Micropressure wave gradient calculation.**

[6] gives a way to calculate approximate maximum pressure wave gradients, using the following method:

Variables used in calculation:

Symbol	Variable	Unit in equations
P	Atmospheric pressure (100 kPa)	Pa
T	Tunnel air temperature (20°C)	°C
V	Speed of train	m/s
k	Nose loss coefficient (aerodynamic property, estimated at 0.05)	none
A_0	Train cross-sectional area	m^2
D	Diameter of tunnel	m
Φ	Tunnel entry length coefficient (Used if no entry hood; distance at entrance over which pressure wave develops; estimated at 1.20)	none
L_T	Length of tunnel	m
Ω	Solid angle available at exit portal of tunnel for pressure wave emission (assumed 4.00)	Steradians (the SI unit of measurement of surface fraction of a sphere)
r	Measurement distance from tunnel portal (20m)	m
L_H	Entry hood length (if used)	m
η	Entry hood efficiency (if used, estimated at 0.70)	none
A_1	Cross-sectional area of tunnel (calculated)	m^2
ρ	Density of air (calculated)	kg/m^3
c	Speed of sound in air (calculated)	m/s
Δt_{rise}	Pressure wave rise time (output)	s
Δt_{steep}	Pressure wave steepening time (output)	s

$\frac{dp}{dt}$	Maximum pressure wave gradient at exit portal	
$\frac{dp_{mpw}}{dt}$	Maximum pressure wave gradient at measurement distance (output)	Pa/s
Δp_{mpw}	Maximum pressure wave amplitude at measurement distance (Not featured in [6]; output)	

Calculation steps:

To find air density

$$\rho = \frac{P}{287(T + 273)}$$

Speed of sound in the tunnel

$$c = 1.4287(T + 273)$$

Coefficients used in calculation of pressure change

$$\beta = \frac{A_0}{A_1}$$

$$\alpha = \frac{1 + k}{(1 - \beta)^2} - 1$$

Pressure change

$$\Delta p \ = \rho c \left[V + \frac{c}{\alpha}\left(1 - \sqrt{1 + \frac{2\alpha V}{c}} \right) \right]$$

Effective length – calculated differently depending on existence of entry hood

$$L_E = \Phi D$$

$$or\ L_E = \eta L_H$$

Pressure wave rise time

$$\Delta t_{rise} = L_E \left(\frac{1}{V} - \frac{1}{c}\right)$$

Pressure wave steepening time

$$\Delta t_{steep} = \frac{1.2 \Delta p L_T}{\rho c^3}$$

Max pressure wave gradient

$$\frac{dp}{dt} = \frac{\Delta p}{\Delta t_{rise} - \Delta t_{steep}}$$

Max pressure wave gradient at measurement position

$$\frac{dp_{mpw}}{dt} = \frac{2A_1}{\Omega rc}\frac{dp}{dt}$$

Max pressure change at measurement position

$$\Delta p_{mpw} = \frac{2A_1}{\Omega rc}\Delta \mathrm{p}$$

Example of spreadsheet calculation:

Variables	Unit	Value					
Input							
Atmospheric Pressure	Pa	100,000.00	100,000.00	100,000.00	100,000.00	100,000.00	100,000.00
Ambient Temperature	°C	20.00	20.00	20.00	20.00	20.00	20.00
Speed of Train	m/s	100.00	100.00	100.00	100.00	100.00	100.00
Nose Loss Coefficient		0.05	0.05	0.05	0.05	0.05	0.05
Cross-Sectional Area of Train	m²	12.00	12.00	12.00	12.00	12.00	12.00
Diameter of Tunnel	m	14.00	14.00	7.00	7.00	7.00	7.00
Entry Length Coefficient		1.20	1.20	1.20	1.20	1.20	1.20
Entry Hood Efficiency		0.70	0.70	0.70	0.70	0.70	0.70
Length of Entry Hood	m	0.00	0.00	0.00	0.00	0.00	0.00
Length of Tunnel	m	1,800.00	2,700.00	8,200.00	24,500.00	26,000.00	30,200.00
Micro-Pressure Wave Emission Angle	Steradians	4.00	4.00	4.00	4.00	4.00	4.00
Distance from Tunnel Portal	m	20.00	20.00	20.00	20.00	20.00	20.00
Derived							
Air Density	kg/m³	1.19	1.19	1.19	1.19	1.19	1.19
Speed of Sound in Air	m/s	343.11	343.11	343.11	343.11	343.11	343.11
Blockage Ratio		0.08	0.08	0.31	0.31	0.31	0.31
Cross-Sectional Area of Tunnel	m²	153.86	153.86	38.47	38.47	38.47	38.47
Calculations							
Alpha		0.2351532466	0.2351532466	1.218078826	1.218078826	1.218078826	1.218078826
Entry Pressure Change	Pa	1309.875661	1309.875661	5440.096941	5440.096941	5440.096941	5440.096941
Tunnel Entry Length	m	16.8	16.8	8.4	8.4	8.4	8.4
Pressure Wave Rise Time	s	0.1190367197	0.1190367197	0.05951835987	0.05951835987	0.05951835987	0.05951835987
Entry Pressure Gradient	Pa/s	11003.96301	11003.96301	91401.99684	91401.99684	91401.99684	91401.99684
Pressure Wave Steepening Time	s	0.06	0.09	1.11	3.33	3.53	4.10
Exit Pressure Gradient	Pa/s	21,781.71	42,685.91	-5,157.15	-1,663.62	-1,565.99	-1,345.00
Maximum Pressure Wave Gradient	Pa/s	244.1849985	478.5325779	-14.4536123	-4.662512483	-4.388912744	-3.769552288
Maximum Pressure Wave Magnitude	Pa/s	14.68442796	14.68442796	15.24662111	15.24662111	15.24662111	15.24662111

11. PRESSURE CONVERSION

Sound pressure (p, Pa) can be converted to sound pressure level (L_p, dB) by the following relation:

$$L_p = 20 \cdot \log_{10}\left(\frac{p}{p_0}\right)$$

Where $p_0 = 2 \times 10^5$ Pa.

12. ULD CONTAINER BAG CAPACITY

The average bag allowance for standard economy class passengers is one 23kg item of luggage with the sum of all three dimensions less than or equal to 158 cm. The maximum number of passengers per carriage is 57.

The maximum volume that can be taken by a bag is attained when all sides are 52.67 cm. This is a direct result of the AM-GM inequality, which states that for all real a, b and c,

$$\frac{a+b+c}{3} \geq \sqrt[3]{abc}$$

Or

$$abc \leq \left(\frac{a}{3} + \frac{b}{3} + \frac{c}{3}\right)^3$$

With equality when a, b and c are equal. This implies that volume of a bag, abc, is maximised when the lengths a, b and c are all equal to one third of 158 cm. This maximum volume per bag is then $\left(\frac{1.58}{3}\right)^3 = 0.146\ m^3$. Therefore, each carriage should be able to hold at least 57 times this, $8.33\ m^3$, and should have spare room for awkwardly-shaped bags.

13. CONGESTION ESTIMATIONS

The congestion as provided at the start of the challenge is reproduced in the following table.

ROUTE	CONGESTION - PEAK	CONGESTION - HOURLY AVERAGE
G-S	510	430
G-C	310	260
G-H	1200	950
G-L	670	510
S-C	220	170
S-H	780	600
S-L	330	260
C-H	670	510
C-L	220	170
H-L	1100	850

Most useful here is the hourly average congestion. However, the congestion across a day will be far from static, due to different flight volumes during the night, as well as commuter access to the trains. We can estimate the first of these contributions by assuming that passenger volume at a given time is proportional to flight volume.

CONGESTION RESEARCH

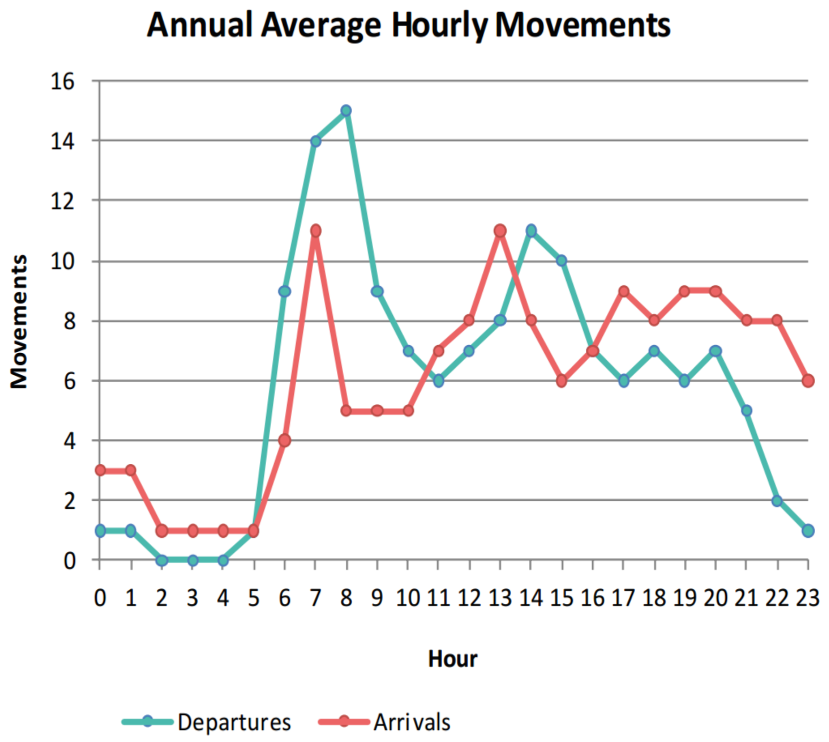
Heathrow, Gatwick and Stansted have flight limits imposed during the night. Luton also has its own limits. City shuts for the majority of the night. Heathrow also has no flights for a period during the night. The limits on flights are based on sound levels in addition to flight numbers, so are subject to change depending on the types of planes used, as different planes have different noise levels.

The following tables summarise estimations [102-108]. Where data is unknown, cells are left blank.

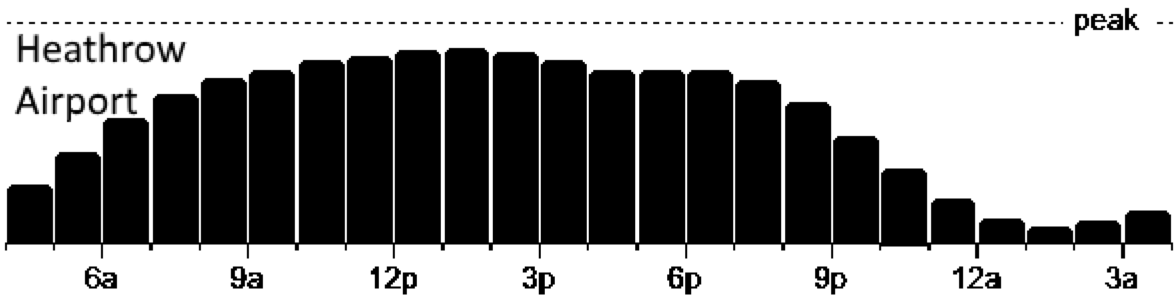
	No-Flight Period Starts	No-Flight Period Ends	Restriction Starts	Restriction End	Seasonal Average Night Movement Limit	Daily Average Night Movement Limit	Total Movements Per Day (Average)
Heathrow	23:30	04:30	23:30	06:00	2900	16	1293
Gatwick			23:30	06:00	7225	40	755
Stansted			23:30	06:00	6000	33	452
Luton			23:30	07:00		28	285
City - Average						5	205
City - Weekdays	22:00	06:30	06:30	07:00		6	592
City - Saturdays	12:30	06:30	06:30	07:00		6	100
City - Sundays	22:00	12:30				0	200

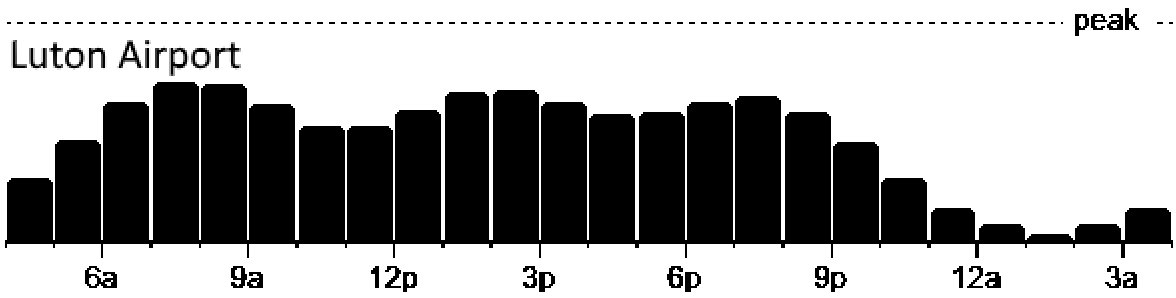
	Length of Restricted Period /hrs	Flights per Hour in Restricted Period	Flights per Hour in Non-Restricted Period
Heathrow	6.5	2.46	72.97
Gatwick	6.5	6.15	40.86
Stansted	6.5	5.08	23.94
Luton	7.5	3.73	15.58
City - Average	0.5	10.00	15.41

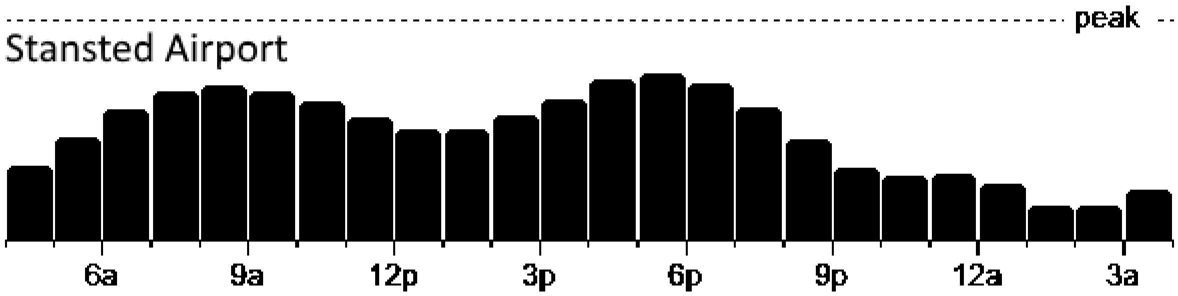
Provided in [109] is a graph of average departure and arrivals per hour, displayed below:

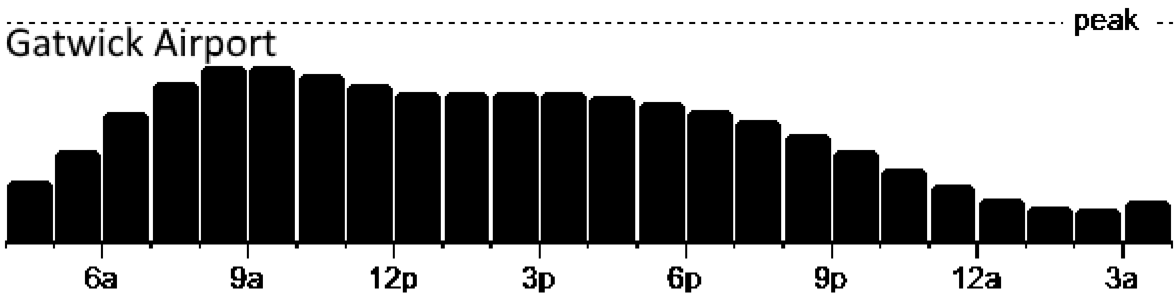


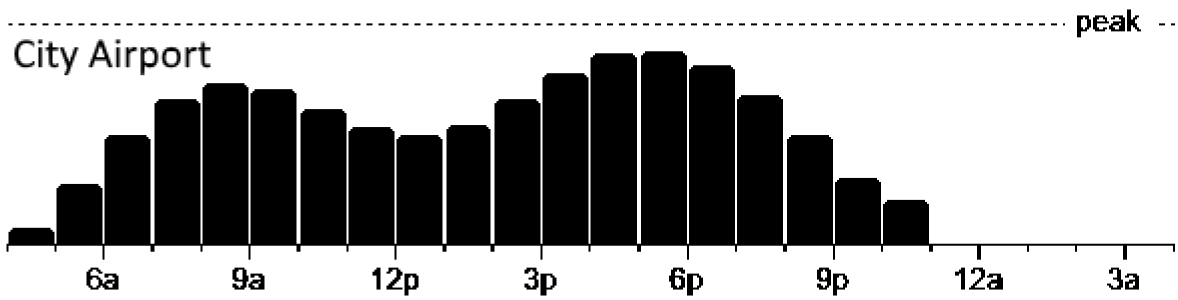
This shape of graph is likely to be characteristic of all the airports. We can compare with Google's crowd-sourced data of congestion at different times in each airport in the following graphs:











The peaks and troughs of the Luton congestion graph approximately line up to those on the arrival and departure graph. It is reasonable to assume that the data even more closely corresponds to the movements of passengers into and out of the airports at different times. Using this assumption, we can estimate congestion over time effectively, if we can fit our known peak and average congestion values to graphs with the form of those shown above.

CALCULATING CONGESTION BY HOUR

Using the given hourly average data, we can find the total passengers leaving and entering an airport in a day, by summing the hourly average contributions from all routes and multiplying by 24. The results are shown in this table:

AIRPORT	TOTAL CONGESTION LEAVING (= TOTAL ARRIVING)
G	51600
S	35040
C	26640
H	69840
L	42960

The numbers arriving and leaving are identical since the given numbers are the same going in both directions on a single route.

With this knowledge, we can find the distribution of passengers across against time through a day for each route. For each route, we:

- Calculate the weighting of passengers due to the source and destination
- Sum the Google congestion data across a day for the source and destination airports
- Find a ratio of these congestion totals which is equal to the ratio of the totals in the given data
- Find the factor to multiply the sum of these weighted totals by to get back the original total
- Weight the time distribution data with these factors
- Sum them to find the overall time distribution for that route.

The weighting calculation is done as follows, where

- T_S and T_D are the given source and destination total congestions
- T_S' and T_D' are the totals from the Google congestion data
- T is the total congestion for the route
- n_S^i and n_D^i are the unweighted source and destination congestions from Google data in hour i.

$$\frac{T_S}{T_D} = \alpha \cdot \frac{T_S'}{T_D'}$$

$$\Rightarrow \alpha = \frac{T_D' \cdot T_S}{T_S' \cdot T_D}$$

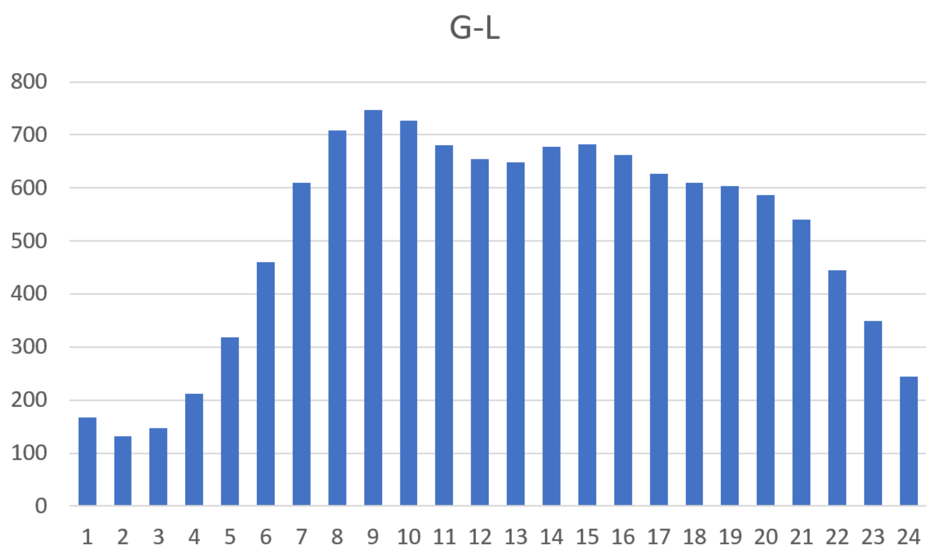
$$\beta(\alpha \cdot T_S' + T_D') = T$$

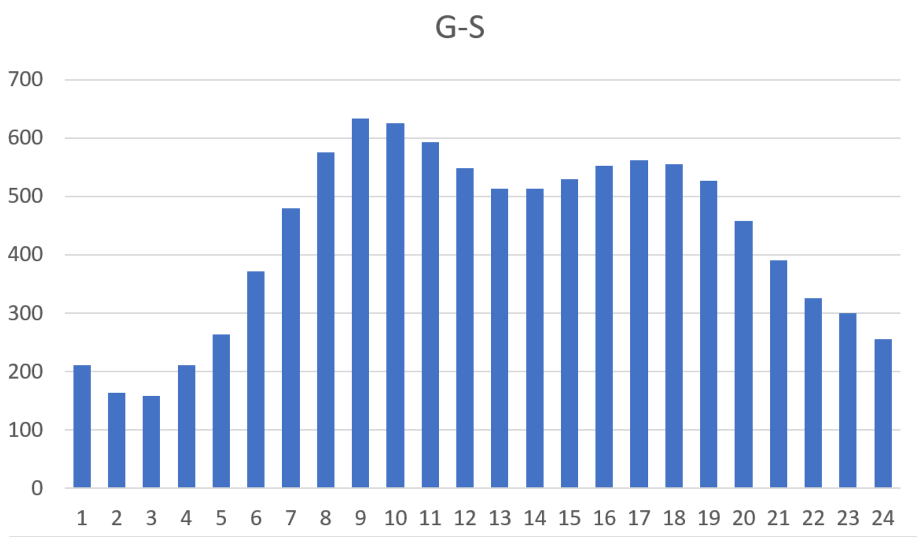
$$\Rightarrow \beta = \frac{T}{\alpha \cdot T_S' + T_D'}$$

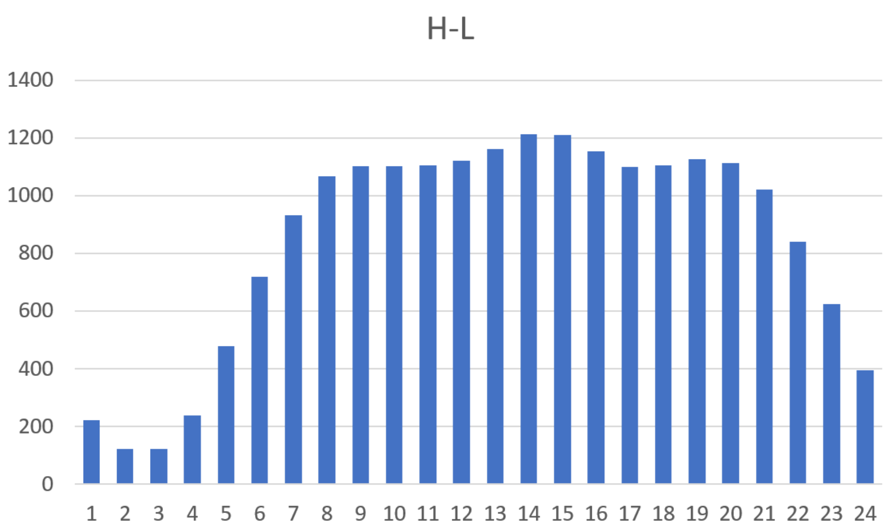
For each hour, we can then find the total congestion N^i in that hour by

$$N^i = \beta\left(\alpha \cdot n_S^i + n_D^i\right)$$

Results are shown the following table and example graphs:







	TOTAL	ALPHA	BETA	FACTOR	0.00	1.00	2.00	3.00	4.00	5.00	6.00	7.00	8.00	9.00	10.00	11.00	12.00	13.00	14.00	15.00	16.00	17.00	18.00	19.00	20.00	21.00	22.00	23.00	SUM
G-S	10320	1.223	40.094	6146	211	164	159	211	263	371	480	576	633	625	593	549	513	513	529	553	563	555	527	459	390	325	301	256	10320
G-C	6240	1.861	17.647	4115	82	72	69	82	99	159	240	318	386	404	382	348	321	314	322	344	360	374	366	335	289	241	189	146	6240
G-H	22800	0.808	95.640	9688	375	275	239	327	500	744	967	1163	1298	1326	1334	1314	1302	1312	1302	1274	1222	1191	1160	1092	987	862	699	536	22800
G-L	12240	1.082	49.254	6679	168	132	146	212	318	461	609	708	747	727	681	654	648	677	682	663	627	610	604	587	541	445	350	244	12240
S-G	10320	0.817	49.052	4174	211	164	159	211	263	371	480	576	633	625	593	549	513	513	529	553	563	555	527	459	390	325	301	256	10320
S-C	4080	1.521	14.636	2318	49	31	31	49	65	104	155	202	236	246	232	206	186	180	196	227	262	291	291	255	205	157	128	94	4080
S-H	14400	0.661	69.942	4811	235	142	121	200	330	486	616	723	774	786	800	791	786	793	805	811	818	832	823	751	651	542	451	337	14400
S-L	6240	0.885	30.441	2803	81	47	59	108	176	251	323	363	359	342	319	305	303	322	336	340	344	355	358	335	293	224	181	119	6240
C-G	6240	0.537	32.844	2125	82	72	69	82	99	159	240	318	386	404	382	348	321	314	322	344	360	374	366	335	289	241	189	146	6240
C-S	4080	0.657	22.265	1762	49	31	31	49	65	104	155	202	236	246	232	206	186	180	196	227	262	291	291	255	205	157	128	94	4080
C-H	12240	0.434	64.627	3380	123	71	52	90	181	306	440	560	655	703	715	699	690	685	693	707	730	772	780	738	645	537	397	271	12240
C-L	4080	0.581	22.306	1562	16	7	16	36	71	118	177	217	236	240	222	209	206	214	222	229	240	261	272	266	235	180	123	68	4080
H-G	22800	1.237	77.316	13112	375	275	239	327	500	744	967	1163	1298	1326	1334	1314	1302	1312	1302	1274	1222	1191	1160	1092	987	862	699	536	22800
H-S	14400	1.513	46.215	9589	235	142	121	200	330	486	616	723	774	786	800	791	786	793	805	811	818	832	823	751	651	542	451	337	14400
H-C	12240	2.302	28.071	8860	123	71	52	90	181	306	440	560	655	703	715	699	690	685	693	707	730	772	780	738	645	537	397	271	12240
H-L	20400	1.339	68.816	12631	223	122	122	239	478	720	933	1067	1101	1102	1104	1122	1161	1212	1210	1154	1099	1106	1127	1113	1021	842	626	396	20400
L-G	12240	0.924	53.306	5561	168	132	146	212	318	461	609	708	747	727	681	654	648	677	682	663	627	610	604	587	541	445	350	244	12240
L-S	6240	1.130	26.928	3437	81	47	59	108	176	251	323	363	359	342	319	305	303	322	336	340	344	355	358	335	293	224	181	119	6240
L-C	4080	1.720	12.971	2518	16	7	16	36	71	118	177	217	236	240	222	209	206	214	222	229	240	261	272	266	235	180	123	68	4080
L-H	20400	0.747	92.127	7769	223	122	122	239	478	720	933	1067	1101	1102	1104	1122	1161	1212	1210	1154	1099	1106	1127	1113	1021	842	626	396	20400

These figures are compliant with the original data, in that the peak congestions in the table are all greater than the given peak congestions for at least 2 consecutive hours, and the average passengers per hour are identical to those given.

COMMUTER CONGESTION

Next, the congestion contributions from commuters must be included. It is difficult to quantify these contributions or even find any data at all. We start by researching rail congestion in London, published by the UK Department for Transport [110], shown below:

Chart 3: Arrivals by time band for central London, 2015
(Rail web table RAI0203)

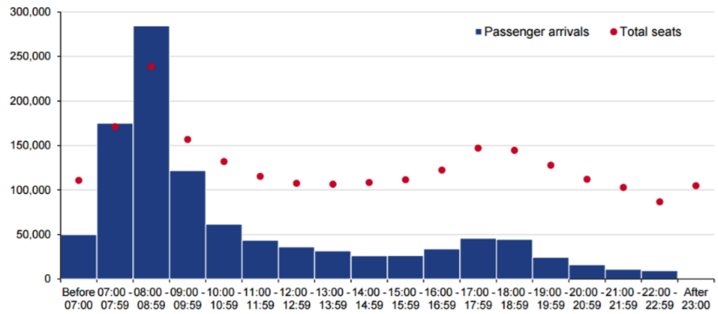
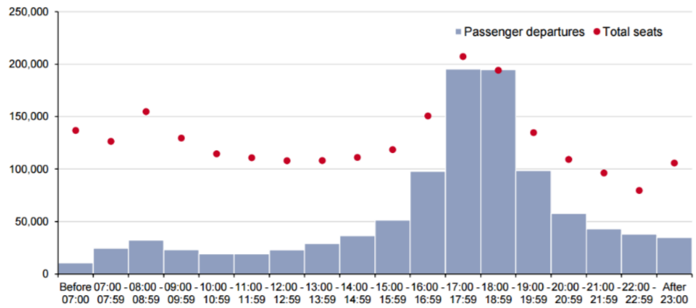


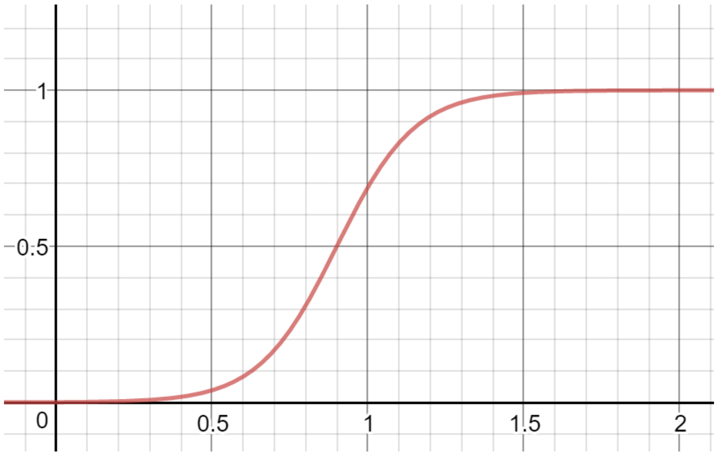
Chart 4: Departures by time band for central London, 2015
(Rail web table RAI0203)



We estimate that:

- 1% of the net travellers into and out of London by rail will switch over to our City lines
- As the ratio of passengers to seats increases, more passengers use our City lines, up to a maximum of 5%.
- The weighting function for the latter percentage against seat filling ratio is as shown in the graph below:

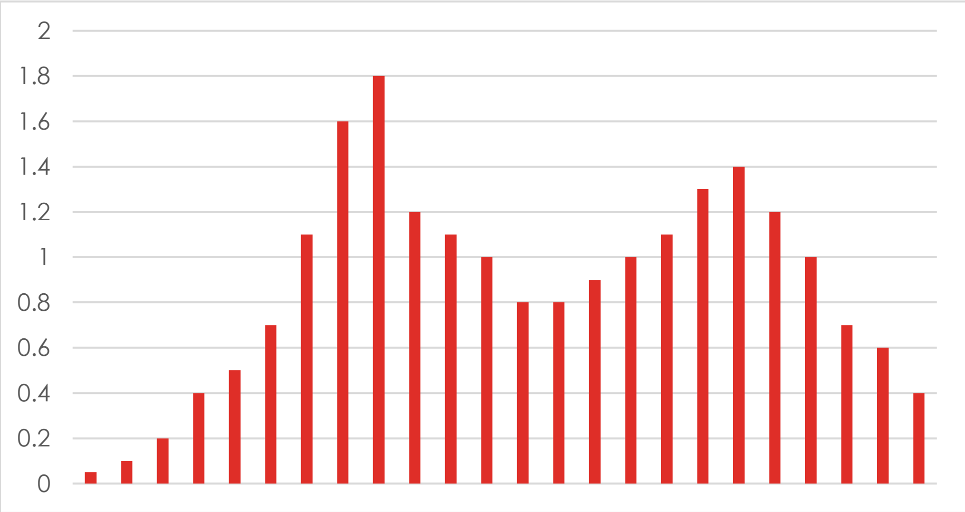
The results of our calculations are shown in the following table.



$$f(x) = \frac{1 + \tanh[4 \cdot (x - 0.9)]}{2}$$

Arrivals		Start of service to 06:59	07:00 - 07:59	08:00 - 08:59	09:00 - 09:59	10:00 - 10:59	11:00 - 11:59	12:00 - 12:59	13:00 - 13:59	14:00 - 14:59	15:00 - 15:59	16:00 - 16:59	17:00 - 17:59	18:00 - 18:59	19:00 - 19:59	20:00 - 20:59	21:00 - 21:59	22:00 - 22:59	23:00 to end of service
London total	Passengers	49988	175144	284344	121863	61583	43799	36313	31737	26267	26457	34035	45844	44697	24440	16169	10994	9576	6693
	Total seats	110775	171115	238579	156877	132004	115412	107503	106494	108389	111525	122437	146954	144574	127695	112013	102963	86602	104852
Departures London total	Passengers	10807	24545	32400	23217	19311	19248	22985	29147	36559	51421	97877	195436	194758	98578	57794	43042	38033	34924
	Total seats	136818	126426	154695	129493	114553	110826	107996	108052	111063	118439	150614	207129	194129	134702	109247	96334	79583	105781
Net Arrivals	Passengers	39181	150599	251944	98647	42272	24550	13328	2590	0	0	0	0	0	0	0	0	0	0
Net Departures	Passengers	0	0	0	0	0	0	0	0	10292	24964	63843	149592	150061	74138	41625	32047	28457	28231
Expected	Into City	392	1613	4148	986	423	246	133	26	0	0	0	0	0	0	0	0	0	0
	Out of City	0	0	0	0	0	0	0	0	103	250	638	1496	1516	741	416	320	285	282
Expected	Weighting	Time																	
Route		< 7:00	07:00	08:00	09:00	10:00	11:00	12:00	13:00	14:00	15:00	16:00	17:00	18:00	19:00	20:00	21:00	22:00	>23:00
G-C	0.340	93	384	987	235	101	58	32	6	0	0	0	0	0	0	0	0	0	0
S-C	0.432	118	488	1254	298	128	74	40	8	0	0	0	0	0	0	0	0	0	0
H-C	0.276	76	312	801	191	82	47	26	5	0	0	0	0	0	0	0	0	0	0
L-C	0.383	105	432	1112	264	113	66	36	7	0	0	0	0	0	0	0	0	0	0
CH-C	0.300	118	484	1244	296	127	74	40	8	0	0	0	0	0	0	0	0	0	0
C-G	0.340	0	0	0	0	0	0	0	0	24	59	152	356	361	176	99	76	68	67
C-S	0.432	0	0	0	0	0	0	0	0	31	75	193	452	458	224	126	97	86	85
C-H	0.276	0	0	0	0	0	0	0	0	20	48	123	289	293	143	80	62	55	55
C-L	0.383	0	0	0	0	0	0	0	0	28	67	171	401	406	199	112	86	76	76
C-CH	0.300	0	0	0	0	0	0	0	0	31	75	192	449	455	222	125	96	85	85

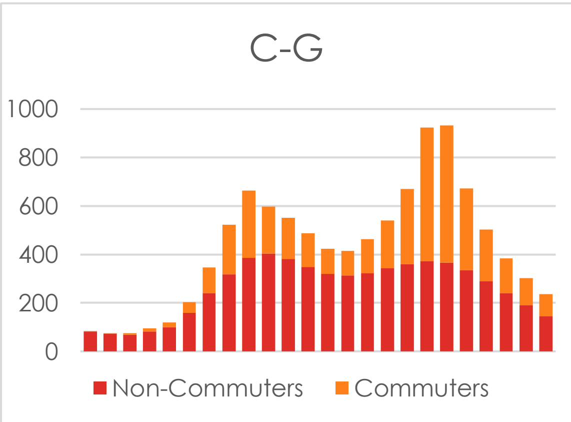
It is more difficult to estimate to additional congestion from commuters on our routes in general. We assume that the number of commuters is 40% of the number of aircraft users for each hour, weighted by the distribution below:

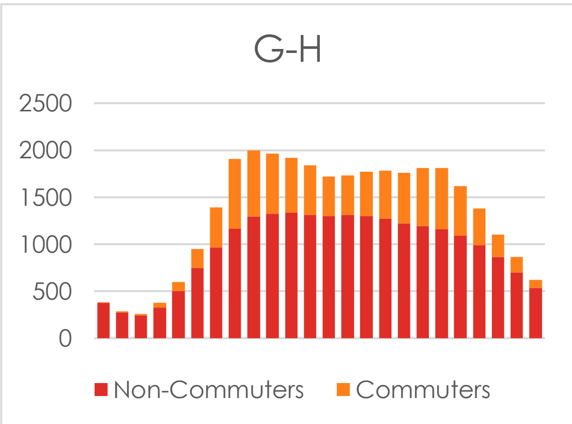


We estimate the number of passengers using the Central Hub lines to be, for each line, an additional 20% of the non-commuter congestion moving out of the other airport the line is connected to.

The number of commuters is limited by our maximum capacity, which is $3 \times 57 \times 12 \approx 2000$ passengers in a single hour.

Combining all this passenger data, we obtain the following table, with example graphs of routes shown:





Route	Time (hours)																						Max (No Commuters)	Given Maxima
	00:00	01:00	02:00	03:00	04:00	05:00	06:00	07:00	08:00	09:00	10:00	11:00	12:00	13:00	14:00	15:00	16:00	17:00	18:00	22:00	23:00	Max		
G-S	211	171	172	245	316	475	691	944	1089	925	854	768	677	677	720	774	810	843	823	373	297	1089	633	510
G-C	82	75	74	95	118	203	439	906	1652	833	651	545	455	420	438	481	519	568	570	234	169	1652	404	310
G-H	375	286	258	380	600	952	1392	1907	2000	1963	1920	1840	1719	1732	1771	1783	1759	1810	1809	867	622	2000	1334	1200
G-L	168	137	158	246	381	590	877	1161	1284	1076	981	916	855	894	928	928	903	928	942	433	283	1284	747	670
G-CH	0	13	25	67	118	243	505	885	1103	740	658	573	445	451	510	567	610	710	744	185	95	1103	n/a	n/a
S-G	211	171	172	245	316	475	691	944	1089	925	854	768	677	677	720	774	810	843	823	373	297	1089	633	510
S-C	49	32	34	57	77	133	342	819	1660	663	462	363	286	245	267	318	378	442	454	158	110	1660	291	220
S-H	235	147	130	232	396	622	887	1185	1331	1163	1152	1107	1038	1047	1094	1136	1178	1265	1283	559	391	1331	832	780
S-L	81	49	64	125	211	321	466	596	618	506	459	427	401	425	456	475	495	539	559	224	138	618	363	330
S-CH	0	8	15	45	83	170	346	597	721	480	428	370	286	289	336	386	437	528	560	127	65	721	n/a	n/a
C-G	82	75	74	95	118	203	346	522	664	598	550	487	423	414	463	540	671	924	931	302	236	931	404	310
C-S	49	32	34	57	77	133	224	332	406	364	334	289	245	238	298	393	571	895	912	244	195	912	291	220
C-H	123	74	56	105	217	392	633	919	1127	1040	1029	979	911	904	962	1038	1174	1462	1510	547	368	1510	780	670
C-L	16	7	17	41	86	151	255	355	406	355	320	293	271	282	330	388	516	798	831	229	155	831	272	220
C-CH	0	4	7	21	42	96	223	415	545	382	341	293	224	223	258	301	350	441	478	100	46	545	n/a	n/a
H-G	375	286	258	380	600	952	1392	1907	2000	1963	1920	1840	1719	1732	1771	1783	1759	1810	1809	867	622	2000	1334	1200
H-S	235	147	130	232	396	622	887	1185	1331	1163	1152	1107	1038	1047	1094	1136	1178	1265	1283	559	391	1331	832	780
H-C	123	74	56	105	217	392	709	1231	1929	1231	1111	1027	936	909	942	990	1051	1173	1217	492	314	1929	780	670
H-L	223	127	132	277	574	921	1344	1750	1894	1630	1590	1571	1533	1600	1645	1616	1583	1681	1758	776	459	1894	1212	1100
H-CH	0	12	21	69	149	316	650	1124	1378	940	869	785	630	640	722	789	851	1014	1089	261	123	1378	n/a	n/a
L-G	168	137	158	246	381	590	877	1161	1284	1076	981	916	855	894	928	928	903	928	942	433	283	1284	747	670
L-S	81	49	64	125	211	321	466	596	618	506	459	427	401	425	456	475	495	539	559	224	138	618	363	330
L-C	16	7	17	41	86	151	360	787	1518	619	433	359	307	289	303	321	345	397	424	153	79	1518	272	220
L-H	223	127	132	277	574	921	1344	1750	1894	1630	1590	1571	1533	1600	1645	1616	1583	1681	1758	776	459	1894	1212	1100
L-CH	0	6	14	48	104	217	449	753	880	578	512	458	371	388	441	477	508	606	661	154	66	880	n/a	n/a
CH-G	0	13	25	67	118	243	505	885	1103	740	658	573	445	451	510	567	610	710	744	185	95	1103	n/a	n/a
CH-S	0	8	15	45	83	170	346	597	721	480	428	370	286	289	336	386	437	528	560	127	65	721	n/a	n/a
CH-C	0	4	7	21	42	96	223	415	545	382	341	293	224	223	258	301	350	441	478	100	46	545	n/a	n/a
CH-H	0	12	21	69	149	316	650	1124	1378	940	869	785	630	640	722	789	851	1014	1089	261	123	1378	n/a	n/a
CH-L	0	6	14	48	104	217	449	753	880	578	512	458	371	388	441	477	508	606	661	154	66	880	n/a	n/a

We wish to stick to the given constraint of having a maximum waiting time of 5 minutes as much as possible.

To achieve this, whilst simultaneously maximising efficiency, we can vary the number of carriages on the trains in service during their 10 minute stops at stations.

By adding carriages to only some of the four carriages in service at any given time to meet demand, we can allow average carriage capacity steps of 0.25 carriages. By changing carriage number up to 4, we can further decrease our average carriage "step size" per hour to 0.0625.

Using these techniques allows us to closely match demand and deliver 12 trains an hour as long as the demand is sufficient.

However, we require at least 1 carriage every 5 minutes to meet this criterion. The congestion is very low in some periods, down to 0 passengers an hour on some routes at night, and 245 passengers an hour even on main routes at midday. If we were to have 12 trains per hour during a period such as the latter, there would be about 20 people per train – about 36% of the train's capacity. During the night, when there are very few passengers, this number would be even lower.

The majority of energy costs in our system are due to work done against air resistance. This is completely independent of mass, and therefore of the number of people on a train. As such, it is extremely inefficient and environmentally unfriendly to run 12 trains an hour in periods of low congestion.

In addition, at night – particularly from 1:00 to 2:59 – there are few passengers across all stations and routes. If we do not shut down our system in this period, we will have to pay station staff for very little relative profit, and we will have no time to perform maintenance and cleaning on both the trains and stations.

With these considerations taken into account, **we will choose not to stick to the guideline of having a waiting time of less than 5 minutes at any time of day, in favour of cost and environmental benefits, as well as time for maintenance of trains**.

We will also have longer waiting times during quiet periods. We will still, however, have the maximum sensible number of trains per hour possible.

Directly calculated, the maximum possible number of trains each hour that does not exceed the expected number of passengers for that hour, and associated average carriage numbers, are given in the following tables:

Route	Time (hours)																							
	0.00	1.00	2.00	3.00	4.00	5.00	6.00	7.00	8.00	9.00	10.00	11.00	12.00	13.00	14.00	15.00	16.00	17.00	18.00	19.00	20.00	21.00	22.00	23.00
G-S	1.00	1.00	1.00	1.00	1.00	1.00	1.06	1.44	1.63	1.38	1.25	1.13	1.00	1.00	1.06	1.19	1.19	1.25	1.25	1.00	1.00	1.00	1.00	1.00
G-C	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.38	2.44	1.25	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
G-H	1.00	1.00	1.00	1.00	1.00	1.44	2.06	2.81	2.94	2.88	2.81	2.75	2.56	2.56	2.63	2.63	2.63	2.69	2.69	2.38	2.06	1.63	1.31	1.00
G-L	1.00	1.00	1.00	1.00	1.00	1.00	1.31	1.75	1.94	1.63	1.44	1.38	1.31	1.31	1.38	1.38	1.38	1.38	1.44	1.31	1.13	1.00	1.00	1.00
G-CH	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.31	1.63	1.13	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.06	1.13	1.00	1.00	1.00	1.00	1.00
S-G	1.00	1.00	1.00	1.00	1.00	1.00	1.06	1.44	1.63	1.38	1.25	1.13	1.00	1.00	1.06	1.19	1.19	1.25	1.25	1.00	1.00	1.00	1.00	1.00
S-C	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.25	2.44	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
S-H	1.00	1.00	1.00	1.00	1.00	1.00	1.31	1.75	2.00	1.75	1.69	1.63	1.56	1.56	1.63	1.69	1.75	1.88	1.94	1.63	1.38	1.06	1.00	1.00
S-L	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
S-CH	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.06	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
C-G	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.38	1.38	1.00	1.00	1.00	1.00	1.00
C-S	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.31	1.38	1.00	1.00	1.00	1.00	1.00
C-H	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.38	1.69	1.56	1.56	1.44	1.38	1.38	1.44	1.56	1.75	2.19	2.25	1.81	1.50	1.13	1.00	1.00
C-L	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.19	1.25	1.00	1.00	1.00	1.00	1.00
C-CH	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
H-G	1.00	1.00	1.00	1.00	1.00	1.44	2.06	2.81	2.94	2.88	2.81	2.75	2.56	2.56	2.63	2.63	2.63	2.69	2.69	2.38	2.06	1.63	1.31	1.00
H-S	1.00	1.00	1.00	1.00	1.00	1.00	1.31	1.75	2.00	1.75	1.69	1.63	1.56	1.56	1.63	1.69	1.75	1.88	1.94	1.63	1.38	1.06	1.00	1.00
H-C	1.00	1.00	1.00	1.00	1.00	1.00	1.06	1.81	2.88	1.81	1.63	1.56	1.38	1.38	1.44	1.50	1.56	1.75	1.81	1.63	1.38	1.06	1.00	1.00
H-L	1.00	1.00	1.00	1.00	1.00	1.38	2.00	2.56	2.81	2.44	2.38	2.31	2.25	2.38	2.44	2.38	2.38	2.50	2.63	2.44	2.13	1.63	1.19	1.00
H-CH	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.69	2.06	1.38	1.31	1.19	1.00	1.00	1.06	1.19	1.25	1.50	1.63	1.31	1.00	1.00	1.00	1.00
L-G	1.00	1.00	1.00	1.00	1.00	1.00	1.31	1.75	1.94	1.63	1.44	1.38	1.31	1.31	1.38	1.38	1.38	1.38	1.44	1.31	1.13	1.00	1.00	1.00
L-S	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
L-C	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.19	2.25	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
L-H	1.00	1.00	1.00	1.00	1.00	1.38	2.00	2.56	2.81	2.44	2.38	2.31	2.25	2.38	2.44	2.38	2.38	2.50	2.63	2.44	2.13	1.63	1.19	1.00
L-CH	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.13	1.31	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
CH-G	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.31	1.63	1.13	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.06	1.13	1.00	1.00	1.00	1.00	1.00
CH-S	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.06	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
CH-C	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
CH-H	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.69	2.06	1.38	1.31	1.19	1.00	1.00	1.06	1.19	1.25	1.50	1.63	1.31	1.00	1.00	1.00	1.00
CH-L	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.13	1.31	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00

Route	Time (hours)																							
	0.00	1.00	2.00	3.00	4.00	5.00	6.00	7.00	8.00	9.00	10.00	11.00	12.00	13.00	14.00	15.00	16.00	17.00	18.00	19.00	20.00	21.00	22.00	23.00
G-S	4	3	4	5	6	9	12	12	12	12	12	12	12	12	12	12	12	12	12	12	10	8	7	6
G-C	2	2	2	2	3	4	8	12	12	12	12	10	8	8	8	9	10	10	11	9	8	6	5	3
G-H	7	6	5	7	11	12	12	12	12	12	12	12	12	12	12	12	12	12	12	12	12	12	12	11
G-L	4	3	3	5	7	11	12	12	12	12	12	12	12	12	12	12	12	12	12	12	12	10	8	5
G-CH	1	1	1	2	3	5	9	12	12	12	12	11	8	8	9	10	11	12	12	11	8	5	4	2
S-G	4	3	4	5	6	9	12	12	12	12	12	12	12	12	12	12	12	12	12	12	10	8	7	6
S-C	1	1	1	1	2	3	7	12	12	12	9	7	6	5	5	6	7	8	8	7	6	4	3	2
S-H	5	3	3	5	7	11	12	12	12	12	12	12	12	12	12	12	12	12	12	12	12	12	10	7
S-L	2	1	2	3	4	6	9	11	11	9	9	8	8	8	9	9	9	10	10	9	8	6	4	3
S-CH	1	1	1	1	2	3	7	11	12	9	8	7	6	6	6	7	8	10	10	8	6	4	3	2
C-G	2	2	2	2	3	4	7	10	12	11	10	9	8	8	9	10	12	12	12	12	9	7	6	5
C-S	1	1	1	1	2	3	4	6	8	7	6	6	5	5	6	7	11	12	12	11	8	6	5	4
C-H	3	2	1	2	4	7	12	12	12	12	12	12	12	12	12	12	12	12	12	12	12	12	10	7
C-L	1	1	1	1	2	3	5	7	8	7	6	6	5	5	6	7	10	12	12	11	8	6	5	3
C-CH	1	1	1	1	1	2	4	8	10	7	6	6	4	4	5	6	7	8	9	7	5	3	2	1
H-G	7	6	5	7	11	12	12	12	12	12	12	12	12	12	12	12	12	12	12	12	12	12	12	11
H-S	5	3	3	5	7	11	12	12	12	12	12	12	12	12	12	12	12	12	12	12	12	12	10	7
H-C	3	2	1	2	4	7	12	12	12	12	12	12	12	12	12	12	12	12	12	12	12	12	9	6
H-L	4	3	3	5	11	12	12	12	12	12	12	12	12	12	12	12	12	12	12	12	12	12	12	9
H-CH	1	1	1	2	3	6	12	12	12	12	12	12	12	12	12	12	12	12	12	12	12	7	5	3
L-G	4	3	3	5	7	11	12	12	12	12	12	12	12	12	12	12	12	12	12	12	12	10	8	5
L-S	2	1	2	3	4	6	9	11	11	9	9	8	8	8	9	9	9	10	10	9	8	6	4	3
L-C	1	1	1	1	2	3	7	12	12	11	8	7	6	6	6	6	7	7	8	7	6	5	3	2
L-H	4	3	3	5	11	12	12	12	12	12	12	12	12	12	12	12	12	12	12	12	12	12	12	9
L-CH	1	1	1	1	2	4	8	12	12	11	9	9	7	7	8	9	9	11	12	10	8	5	3	2
CH-G	1	1	1	2	3	5	9	12	12	12	12	11	8	8	9	10	11	12	12	11	8	5	4	2
CH-S	1	1	1	1	2	3	7	11	12	9	8	7	6	6	6	7	8	10	10	8	6	4	3	2
CH-C	1	1	1	1	1	2	4	8	10	7	6	6	4	4	5	6	7	8	9	7	5	3	2	1
CH-H	1	1	1	2	3	6	12	12	12	12	12	12	12	12	12	12	12	12	12	12	12	7	5	3
CH-L	1	1	1	1	2	4	8	12	12	11	9	9	7	7	8	9	9	11	12	10	8	5	3	2

This table needs to be adjusted; we need to remove the trains between 1:00 and 2:59, and we will round the number of trains up to the next highest multiple of 3 if it is already above 4. The number of trains on one route also needs to be equal to the number on the same route in the opposite direction. Taking the greater of both routes for each pair gives us a final timetable of:

Route	Time (hours)																							
	0.00	1.00	2.00	3.00	4.00	5.00	6.00	7.00	8.00	9.00	10.00	11.00	12.00	13.00	14.00	15.00	16.00	17.00	18.00	19.00	20.00	21.00	22.00	23.00
G-S	4	0	0	6	6	9	12	12	12	12	12	12	12	12	12	12	12	12	12	12	12	9	9	6
G-C	2	0	0	2	4	4	9	12	12	12	12	12	9	9	9	9	12	12	12	9	9	6	6	4
G-H	9	0	0	9	12	12	12	12	12	12	12	12	12	12	12	12	12	12	12	12	12	12	12	12
G-L	4	0	0	6	9	12	12	12	12	12	12	12	12	12	12	12	12	12	12	12	12	12	9	6
G-CH	2	0	0	2	4	6	9	12	12	12	12	12	9	9	9	12	12	12	12	12	9	6	4	2
S-G	4	0	0	6	6	9	12	12	12	12	12	12	12	12	12	12	12	12	12	12	12	9	9	6
S-C	2	0	0	2	2	4	9	12	12	12	9	9	6	6	6	6	9	9	9	9	6	4	4	2
S-H	6	0	0	6	9	12	12	12	12	12	12	12	12	12	12	12	12	12	12	12	12	12	12	9
S-L	2	0	0	4	4	6	9	12	12	9	9	9	9	9	9	9	9	12	12	9	9	6	4	4
S-CH	2	0	0	2	2	4	9	12	12	9	9	9	6	6	6	9	9	12	12	9	6	4	4	2
C-G	2	0	0	2	4	4	9	12	12	12	12	9	9	9	9	12	12	12	12	12	9	9	6	6
C-S	2	0	0	2	2	4	4	6	9	9	6	6	6	6	6	9	12	12	12	12	9	6	6	4
C-H	4	0	0	2	4	9	12	12	12	12	12	12	12	12	12	12	12	12	12	12	12	12	12	9
C-L	2	0	0	2	2	4	6	9	9	9	6	6	6	6	6	9	12	12	12	12	9	6	6	4
C-CH	2	0	0	2	2	2	4	9	12	9	6	6	4	4	6	6	9	9	9	9	6	4	2	2
H-G	9	0	0	9	12	12	12	12	12	12	12	12	12	12	12	12	12	12	12	12	12	12	12	12
H-S	6	0	0	6	9	12	12	12	12	12	12	12	12	12	12	12	12	12	12	12	12	12	12	9
H-C	4	0	0	2	4	9	12	12	12	12	12	12	12	12	12	12	12	12	12	12	12	12	9	6
H-L	4	0	0	6	12	12	12	12	12	12	12	12	12	12	12	12	12	12	12	12	12	12	12	9
H-CH	2	0	0	2	4	6	12	12	12	12	12	12	12	12	12	12	12	12	12	12	12	9	6	4
L-G	4	0	0	6	9	12	12	12	12	12	12	12	12	12	12	12	12	12	12	12	12	12	9	6
L-S	2	0	0	4	4	6	9	12	12	9	9	9	9	9	9	9	9	12	12	9	9	6	4	4
L-C	2	0	0	2	2	4	9	12	12	12	9	9	6	6	6	6	9	9	9	9	6	6	4	2
L-H	4	0	0	6	12	12	12	12	12	12	12	12	12	12	12	12	12	12	12	12	12	12	12	9
L-CH	2	0	0	2	2	4	9	12	12	12	9	9	9	9	9	9	9	12	12	12	9	6	4	2
CH-G	2	0	0	2	4	6	9	12	12	12	12	12	9	9	9	12	12	12	12	12	9	6	4	2
CH-S	2	0	0	2	2	4	9	12	12	9	9	9	6	6	6	9	9	12	12	9	6	4	4	2
CH-C	2	0	0	2	2	2	4	9	12	9	6	6	4	4	6	6	9	9	9	9	6	4	2	2
CH-H	2	0	0	2	4	6	12	12	12	12	12	12	12	12	12	12	12	12	12	12	12	9	6	4
CH-L	2	0	0	2	2	4	9	12	12	12	9	9	9	9	9	9	9	12	12	12	9	6	4	2

This gives maximum waiting times between trains as follows:

Route	Time (hours)		In Table: Time between trains (mins:seconds)																					
	0.00	1.00	2.00	3.00	4.00	5.00	6.00	7.00	8.00	9.00	10.00	11.00	12.00	13.00	14.00	15.00	16.00	17.00	18.00	19.00	20.00	21.00	22.00	23.00
G-S	15.00	n/a	n/a	10.00	10.00	6.40	5.00	5.00	5.00	5.00	5.00	5.00	5.00	5.00	5.00	5.00	5.00	5.00	5.00	5.00	5.00	6.40	6.40	10.00
G-C	30.00	n/a	n/a	30.00	15.00	15.00	6.40	5.00	5.00	5.00	5.00	5.00	6.40	6.40	6.40	6.40	5.00	5.00	5.00	6.40	6.40	10.00	10.00	15.00
G-H	6.40	n/a	n/a	6.40	5.00	5.00	5.00	5.00	5.00	5.00	5.00	5.00	5.00	5.00	5.00	5.00	5.00	5.00	5.00	5.00	5.00	5.00	5.00	5.00
G-L	15.00	n/a	n/a	10.00	6.40	5.00	5.00	5.00	5.00	5.00	5.00	5.00	5.00	5.00	5.00	5.00	5.00	5.00	5.00	5.00	5.00	5.00	6.40	10.00
G-CH	n/a	n/a	n/a	30.00	15.00	10.00	6.40	5.00	5.00	5.00	5.00	5.00	6.40	6.40	6.40	5.00	5.00	5.00	5.00	5.00	6.40	10.00	15.00	30.00
S-G	15.00	n/a	n/a	10.00	10.00	6.40	5.00	5.00	5.00	5.00	5.00	5.00	5.00	5.00	5.00	5.00	5.00	5.00	5.00	5.00	5.00	6.40	6.40	10.00
S-C	30.00	n/a	n/a	30.00	30.00	15.00	6.40	5.00	5.00	5.00	6.40	6.40	10.00	10.00	10.00	6.40	5.00	5.00	5.00	5.00	6.40	10.00	10.00	15.00
S-H	10.00	n/a	n/a	10.00	6.40	5.00	5.00	5.00	5.00	5.00	5.00	5.00	5.00	5.00	5.00	5.00	5.00	5.00	5.00	5.00	5.00	5.00	5.00	6.40
S-L	30.00	n/a	n/a	15.00	15.00	10.00	6.40	5.00	5.00	6.40	6.40	6.40	6.40	6.40	6.40	6.40	5.00	5.00	5.00	5.00	6.40	10.00	10.00	15.00
S-CH	n/a	n/a	n/a	30.00	30.00	15.00	6.40	5.00	5.00	6.40	6.40	6.40	10.00	10.00	10.00	6.40	6.40	5.00	5.00	6.40	10.00	15.00	15.00	30.00
C-G	30.00	n/a	n/a	30.00	15.00	15.00	6.40	5.00	5.00	5.00	5.00	5.00	6.40	6.40	6.40	6.40	5.00	5.00	5.00	6.40	6.40	10.00	10.00	15.00
C-S	30.00	n/a	n/a	30.00	30.00	15.00	6.40	5.00	5.00	5.00	6.40	6.40	10.00	10.00	10.00	6.40	5.00	5.00	5.00	5.00	6.40	10.00	10.00	15.00
C-H	15.00	n/a	n/a	30.00	15.00	6.40	5.00	5.00	5.00	5.00	5.00	5.00	5.00	5.00	5.00	5.00	5.00	5.00	5.00	5.00	5.00	5.00	5.00	6.40
C-L	n/a	n/a	n/a	10.00	5.00	5.00	5.00	5.00	5.00	5.00	5.00	5.00	5.00	5.00	5.00	5.00	5.00	5.00	5.00	5.00	5.00	5.00	5.00	6.40
C-CH	n/a	n/a	n/a	30.00	15.00	10.00	5.00	5.00	5.00	5.00	5.00	5.00	5.00	5.00	5.00	5.00	5.00	5.00	5.00	5.00	5.00	6.40	10.00	15.00
H-G	6.40	n/a	n/a	6.40	5.00	5.00	5.00	5.00	5.00	5.00	5.00	5.00	5.00	5.00	5.00	5.00	5.00	5.00	5.00	5.00	5.00	5.00	5.00	5.00
H-S	10.00	n/a	n/a	10.00	6.40	5.00	5.00	5.00	5.00	5.00	5.00	5.00	5.00	5.00	5.00	5.00	5.00	5.00	5.00	5.00	5.00	5.00	5.00	6.40
H-C	15.00	n/a	n/a	30.00	15.00	6.40	5.00	5.00	5.00	5.00	5.00	5.00	5.00	5.00	5.00	5.00	5.00	5.00	5.00	5.00	5.00	5.00	5.00	6.40
H-L	15.00	n/a	n/a	10.00	5.00	5.00	5.00	5.00	5.00	5.00	5.00	5.00	5.00	5.00	5.00	5.00	5.00	5.00	5.00	5.00	5.00	5.00	5.00	6.40
H-CH	n/a	n/a	n/a	30.00	15.00	10.00	5.00	5.00	5.00	5.00	5.00	5.00	5.00	5.00	5.00	5.00	5.00	5.00	5.00	5.00	5.00	6.40	10.00	15.00
L-G	15.00	n/a	n/a	10.00	6.40	5.00	5.00	5.00	5.00	5.00	5.00	5.00	5.00	5.00	5.00	5.00	5.00	5.00	5.00	5.00	5.00	5.00	6.40	10.00
L-S	30.00	n/a	n/a	15.00	15.00	10.00	6.40	5.00	5.00	6.40	6.40	6.40	6.40	6.40	6.40	6.40	5.00	5.00	5.00	5.00	6.40	10.00	10.00	15.00
L-C	n/a	n/a	n/a	10.00	5.00	5.00	5.00	5.00	5.00	5.00	5.00	5.00	5.00	5.00	5.00	5.00	5.00	5.00	5.00	5.00	5.00	5.00	5.00	6.40
L-H	15.00	n/a	n/a	10.00	5.00	5.00	5.00	5.00	5.00	5.00	5.00	5.00	5.00	5.00	5.00	5.00	5.00	5.00	5.00	5.00	5.00	5.00	5.00	6.40
L-CH	n/a	n/a	n/a	30.00	30.00	15.00	6.40	5.00	5.00	5.00	6.40	6.40	6.40	6.40	6.40	6.40	6.40	5.00	5.00	5.00	6.40	10.00	15.00	30.00
CH-G	n/a	n/a	n/a	30.00	15.00	10.00	6.40	5.00	5.00	5.00	5.00	5.00	6.40	6.40	6.40	5.00	5.00	5.00	5.00	5.00	6.40	10.00	15.00	30.00
CH-S	n/a	n/a	n/a	30.00	30.00	15.00	6.40	5.00	5.00	6.40	6.40	6.40	10.00	10.00	10.00	6.40	6.40	5.00	5.00	6.40	10.00	15.00	15.00	30.00
CH-C	n/a	n/a	n/a	30.00	15.00	10.00	5.00	5.00	5.00	5.00	5.00	5.00	5.00	5.00	5.00	5.00	5.00	5.00	5.00	5.00	5.00	6.40	10.00	15.00
CH-H	n/a	n/a	n/a	30.00	15.00	10.00	5.00	5.00	5.00	5.00	5.00	5.00	5.00	5.00	5.00	5.00	5.00	5.00	5.00	5.00	5.00	6.40	10.00	15.00
CH-L	n/a	n/a	n/a	30.00	30.00	15.00	6.40	5.00	5.00	5.00	6.40	6.40	6.40	6.40	6.40	6.40	6.40	5.00	5.00	5.00	6.40	10.00	15.00	30.00

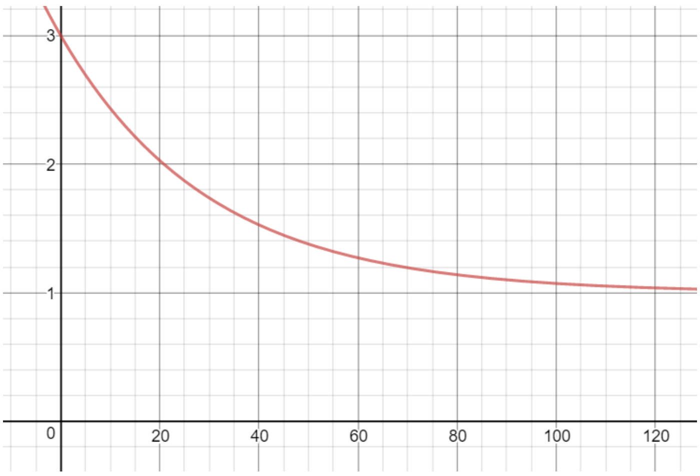
The number of passengers is restricted in some cases by these train numbers, giving the following final table of passenger numbers per hour over a day:

	0.00	1.00	2.00	3.00	4.00	5.00	6.00	7.00	8.00	9.00	10.00	11.00	12.00	13.00	14.00	15.00	16.00	17.00	18.00	19.00	20.00	21.00	22.00	23.00
G-S	215	0	0	245	316	475	691	944	1089	925	854	768	677	677	720	774	810	843	823	679	546	417	373	297
G-C	84	0	0	95	118	203	439	906	1652	833	651	545	455	420	438	481	519	568	570	496	404	308	234	169
G-H	383	0	0	380	600	952	1392	1907	2000	1963	1920	1840	1719	1732	1771	1783	1759	1810	1809	1617	1382	1104	867	622
G-L	171	0	0	246	381	590	877	1161	1284	1076	981	916	855	894	928	928	903	928	942	869	757	570	433	283
G-CH	0	0	0	67	118	243	505	885	1103	740	658	573	445	451	510	567	610	710	744	594	441	262	185	95
S-G	215	0	0	245	316	475	691	944	1089	925	854	768	677	677	720	774	810	843	823	679	546	417	373	297
S-C	50	0	0	57	77	133	342	819	1660	663	462	363	286	245	267	318	378	442	454	377	288	201	158	110
S-H	239	0	0	232	396	622	887	1185	1331	1163	1152	1107	1038	1047	1094	1136	1178	1265	1283	1111	912	694	559	391
S-L	82	0	0	125	211	321	466	596	618	506	459	427	401	425	456	475	495	539	559	496	410	286	224	138
S-CH	0	0	0	45	83	170	346	597	721	480	428	370	286	289	336	386	437	528	560	432	308	175	127	65
C-G	84	0	0	95	118	203	346	522	664	598	550	487	423	414	463	540	671	924	931	672	503	384	302	236
C-S	50	0	0	57	77	133	224	332	406	364	334	289	245	238	298	393	571	895	912	601	414	298	244	195
C-H	125	0	0	105	217	392	633	919	1127	1040	1029	979	911	904	962	1038	1174	1462	1510	1236	984	749	547	368
C-L	0	0	0	41	86	151	255	355	406	355	320	293	271	282	330	388	516	798	831	592	441	317	229	155
C-CH	0	0	0	21	42	96	223	415	545	382	341	293	224	223	258	301	350	441	478	383	275	156	100	46
H-G	383	0	0	380	600	952	1392	1907	2000	1963	1920	1840	1719	1732	1771	1783	1759	1810	1809	1617	1382	1104	867	622
H-S	239	0	0	232	396	622	887	1185	1331	1163	1152	1107	1038	1047	1094	1136	1178	1265	1283	1111	912	694	559	391
H-C	125	0	0	105	217	392	709	1231	1929	1231	1111	1027	936	909	942	990	1051	1173	1217	1093	904	687	492	314
H-L	228	0	0	277	574	921	1344	1750	1894	1630	1590	1571	1533	1600	1645	1616	1583	1681	1758	1647	1429	1077	776	459
H-CH	0	0	0	69	149	316	650	1124	1378	940	869	785	630	640	722	789	851	1014	1089	887	661	390	261	123
L-G	171	0	0	246	381	590	877	1161	1284	1076	981	916	855	894	928	928	903	928	942	869	757	570	433	283
L-S	82	0	0	125	211	321	466	596	618	506	459	427	401	425	456	475	495	539	559	496	410	286	224	138
L-C	0	0	0	41	86	151	360	787	1518	619	433	359	307	289	303	321	345	397	424	394	329	231	153	79
L-H	228	0	0	277	574	921	1344	1750	1894	1630	1590	1571	1533	1600	1645	1616	1583	1681	1758	1647	1429	1077	776	459
L-CH	0	0	0	48	104	217	449	753	880	578	512	458	371	388	441	477	508	606	661	552	418	237	154	66
CH-G	0	0	0	67	118	243	505	885	1103	740	658	573	445	451	510	567	610	710	744	594	441	262	185	95
CH-S	0	0	0	45	83	170	346	597	721	480	428	370	286	289	336	386	437	528	560	432	308	175	127	65
CH-C	0	0	0	21	42	96	223	415	545	382	341	293	224	223	258	301	350	441	478	383	275	156	100	46
CH-H	0	0	0	69	149	316	650	1124	1378	940	869	785	630	640	722	789	851	1014	1089	887	661	390	261	123
CH-L	0	0	0	48	104	217	449	753	880	578	512	458	371	388	441	477	508	606	661	552	418	237	154	66

14. PROFIT ESTIMATIONS

TICKET PRICES BY ROUTE, HOURLY

A sensible option for working out the ticket prices for each route in terms of a single value is to define a price per kilometre. However, the price per kilometre (PPK) should be higher on shorter routes, to prevent ridiculously low prices, and the PPK can be expected to be lower on the longest routes. Therefore, we weight the PPK by the function of route length shown below:



$$f(x) = 1 + 2e^{-\frac{x}{30}}$$

Route	Route	Rounded	Time (hours)																							
	Length /km	Length /km	0.00	1.00	2.00	3.00	4.00	5.00	6.00	7.00	8.00	9.00	10.00	11.00	12.00	13.00	14.00	15.00	16.00	17.00	18.00	19.00	20.00	21.00	22.00	23.00
G-S	93.4	95	£30.90	£ -	£ -	£30.90	£30.90	£30.90	£30.90	£46.40	£46.40	£30.90	£30.90	£30.90	£30.90	£30.90	£30.90	£30.90	£46.40	£46.40	£30.90	£30.90	£30.90	£30.90	£30.90	£30.90
G-C	44.8	45	£19.50	£ -	£ -	£19.50	£19.50	£19.50	£19.50	£29.30	£29.30	£19.50	£19.50	£19.50	£19.50	£19.50	£19.50	£19.50	£29.30	£29.30	£19.50	£19.50	£19.50	£19.50	£19.50	£19.50
G-H	50.5	50	£20.70	£ -	£ -	£20.70	£20.70	£20.70	£20.70	£31.00	£31.00	£20.70	£20.70	£20.70	£20.70	£20.70	£20.70	£20.70	£31.00	£31.00	£20.70	£20.70	£20.70	£20.70	£20.70	£20.70
G-L	106.0	105	£33.40	£ -	£ -	£33.40	£33.40	£33.40	£33.40	£50.10	£50.10	£33.40	£33.40	£33.40	£33.40	£33.40	£33.40	£33.40	£50.10	£50.10	£33.40	£33.40	£33.40	£33.40	£33.40	£33.40
G-CH	63.6	65	£24.00	£ -	£ -	£24.00	£24.00	£24.00	£24.00	£36.00	£36.00	£24.00	£24.00	£24.00	£24.00	£24.00	£24.00	£24.00	£36.00	£36.00	£24.00	£24.00	£24.00	£24.00	£24.00	£24.00
S-G	93.4	95	£30.90	£ -	£ -	£30.90	£30.90	£30.90	£30.90	£46.40	£46.40	£30.90	£30.90	£30.90	£30.90	£30.90	£30.90	£30.90	£46.40	£46.40	£30.90	£30.90	£30.90	£30.90	£30.90	£30.90
S-C	48.6	50	£20.70	£ -	£ -	£20.70	£20.70	£20.70	£20.70	£31.00	£31.00	£20.70	£20.70	£20.70	£20.70	£20.70	£20.70	£20.70	£31.00	£31.00	£20.70	£20.70	£20.70	£20.70	£20.70	£20.70
S-H	103.6	105	£33.40	£ -	£ -	£33.40	£33.40	£33.40	£33.40	£50.10	£50.10	£33.40	£33.40	£33.40	£33.40	£33.40	£33.40	£33.40	£50.10	£50.10	£33.40	£33.40	£33.40	£33.40	£33.40	£33.40
S-L	86.5	85	£28.50	£ -	£ -	£28.50	£28.50	£28.50	£28.50	£42.70	£42.70	£28.50	£28.50	£28.50	£28.50	£28.50	£28.50	£28.50	£42.70	£42.70	£28.50	£28.50	£28.50	£28.50	£28.50	£28.50
S-CH	90.5	90	£29.70	£ -	£ -	£29.70	£29.70	£29.70	£29.70	£44.50	£44.50	£29.70	£29.70	£29.70	£29.70	£29.70	£29.70	£29.70	£44.50	£44.50	£29.70	£29.70	£29.70	£29.70	£29.70	£29.70
C-G	44.8	45	£19.50	£ -	£ -	£19.50	£19.50	£19.50	£19.50	£29.30	£29.30	£19.50	£19.50	£19.50	£19.50	£19.50	£19.50	£19.50	£29.30	£29.30	£19.50	£19.50	£19.50	£19.50	£19.50	£19.50
C-S	48.6	50	£20.70	£ -	£ -	£20.70	£20.70	£20.70	£20.70	£31.00	£31.00	£20.70	£20.70	£20.70	£20.70	£20.70	£20.70	£20.70	£31.00	£31.00	£20.70	£20.70	£20.70	£20.70	£20.70	£20.70
C-H	76.2	75	£26.20	£ -	£ -	£26.20	£26.20	£26.20	£26.20	£39.30	£39.30	£26.20	£26.20	£26.20	£26.20	£26.20	£26.20	£26.20	£39.30	£39.30	£26.20	£26.20	£26.20	£26.20	£26.20	£26.20
C-L	59.1	60	£22.90	£ -	£ -	£22.90	£22.90	£22.90	£22.90	£34.30	£34.30	£22.90	£22.90	£22.90	£22.90	£22.90	£22.90	£22.90	£34.30	£34.30	£22.90	£22.90	£22.90	£22.90	£22.90	£22.90
C-CH	63.1	65	£24.00	£ -	£ -	£24.00	£24.00	£24.00	£24.00	£36.00	£36.00	£24.00	£24.00	£24.00	£24.00	£24.00	£24.00	£24.00	£36.00	£36.00	£24.00	£24.00	£24.00	£24.00	£24.00	£24.00
H-G	50.5	50	£20.70	£ -	£ -	£20.70	£20.70	£20.70	£20.70	£31.00	£31.00	£20.70	£20.70	£20.70	£20.70	£20.70	£20.70	£20.70	£31.00	£31.00	£20.70	£20.70	£20.70	£20.70	£20.70	£20.70
H-S	103.6	105	£33.40	£ -	£ -	£33.40	£33.40	£33.40	£33.40	£50.10	£50.10	£33.40	£33.40	£33.40	£33.40	£33.40	£33.40	£33.40	£50.10	£50.10	£33.40	£33.40	£33.40	£33.40	£33.40	£33.40
H-C	76.2	75	£26.20	£ -	£ -	£26.20	£26.20	£26.20	£26.20	£39.30	£39.30	£26.20	£26.20	£26.20	£26.20	£26.20	£26.20	£26.20	£39.30	£39.30	£26.20	£26.20	£26.20	£26.20	£26.20	£26.20
H-L	55.5	55	£21.80	£ -	£ -	£21.80	£21.80	£21.80	£21.80	£32.70	£32.70	£21.80	£21.80	£21.80	£21.80	£21.80	£21.80	£21.80	£32.70	£32.70	£21.80	£21.80	£21.80	£21.80	£21.80	£21.80
H-CH	11.8	10	£ 7.30	£ -	£ -	£ 7.30	£ 7.30	£ 7.30	£ 7.30	£10.90	£10.90	£ 7.30	£ 7.30	£ 7.30	£ 7.30	£ 7.30	£ 7.30	£ 7.30	£10.90	£10.90	£ 7.30	£ 7.30	£ 7.30	£ 7.30	£ 7.30	£ 7.30
L-G	106.0	105	£33.40	£ -	£ -	£33.40	£33.40	£33.40	£33.40	£50.10	£50.10	£33.40	£33.40	£33.40	£33.40	£33.40	£33.40	£33.40	£50.10	£50.10	£33.40	£33.40	£33.40	£33.40	£33.40	£33.40
L-S	86.5	85	£28.50	£ -	£ -	£28.50	£28.50	£28.50	£28.50	£42.70	£42.70	£28.50	£28.50	£28.50	£28.50	£28.50	£28.50	£28.50	£42.70	£42.70	£28.50	£28.50	£28.50	£28.50	£28.50	£28.50
L-C	59.1	60	£22.90	£ -	£ -	£22.90	£22.90	£22.90	£22.90	£34.30	£34.30	£22.90	£22.90	£22.90	£22.90	£22.90	£22.90	£22.90	£34.30	£34.30	£22.90	£22.90	£22.90	£22.90	£22.90	£22.90
L-H	55.5	55	£21.80	£ -	£ -	£21.80	£21.80	£21.80	£21.80	£32.70	£32.70	£21.80	£21.80	£21.80	£21.80	£21.80	£21.80	£21.80	£32.70	£32.70	£21.80	£21.80	£21.80	£21.80	£21.80	£21.80
L-CH	42.4	40	£18.30	£ -	£ -	£18.30	£18.30	£18.30	£18.30	£27.50	£27.50	£18.30	£18.30	£18.30	£18.30	£18.30	£18.30	£18.30	£27.50	£27.50	£18.30	£18.30	£18.30	£18.30	£18.30	£18.30
CH-G	63.6	65	£24.00	£ -	£ -	£24.00	£24.00	£24.00	£24.00	£36.00	£36.00	£24.00	£24.00	£24.00	£24.00	£24.00	£24.00	£24.00	£36.00	£36.00	£24.00	£24.00	£24.00	£24.00	£24.00	£24.00
CH-S	90.5	90	£29.70	£ -	£ -	£29.70	£29.70	£29.70	£29.70	£44.50	£44.50	£29.70	£29.70	£29.70	£29.70	£29.70	£29.70	£29.70	£44.50	£44.50	£29.70	£29.70	£29.70	£29.70	£29.70	£29.70
CH-C	63.1	65	£24.00	£ -	£ -	£24.00	£24.00	£24.00	£24.00	£36.00	£36.00	£24.00	£24.00	£24.00	£24.00	£24.00	£24.00	£24.00	£36.00	£36.00	£24.00	£24.00	£24.00	£24.00	£24.00	£24.00
CH-H	11.8	10	£ 7.30	£ -	£ -	£ 7.30	£ 7.30	£ 7.30	£ 7.30	£10.90	£10.90	£ 7.30	£ 7.30	£ 7.30	£ 7.30	£ 7.30	£ 7.30	£ 7.30	£10.90	£10.90	£ 7.30	£ 7.30	£ 7.30	£ 7.30	£ 7.30	£ 7.30
CH-L	42.4	40	£18.30	£ -	£ -	£18.30	£18.30	£18.30	£18.30	£27.50	£27.50	£18.30	£18.30	£18.30	£18.30	£18.30	£18.30	£18.30	£27.50	£27.50	£18.30	£18.30	£18.30	£18.30	£18.30	£18.30

Unweighted PPK = 30p

Route	Time (hours)			Profit (Thousands)																					Sum
	0.00	1.00	2.00	3.00	4.00	5.00	6.00	7.00	8.00	9.00	10.00	11.00	12.00	13.00	14.00	15.00	16.00	17.00	18.00	19.00	20.00	21.00	22.00	23.00	Sum
G-S	£ 6.65	£ -	£ -	£ 7.56	£ 9.77	£13.16	£19.15	£39.28	£45.28	£25.62	£23.66	£21.28	£18.76	£18.76	£19.93	£21.45	£33.69	£35.08	£22.78	£18.80	£15.12	£11.54	£10.33	£ 8.24	£ 445.91
G-C	£ 1.63	£ -	£ -	£ 1.86	£ 2.31	£ 3.55	£ 7.67	£23.78	£43.38	£14.55	£11.37	£ 9.53	£ 7.95	£ 7.34	£ 7.66	£ 8.41	£13.62	£14.91	£ 9.97	£ 8.67	£ 7.07	£ 5.38	£ 4.09	£ 2.96	£ 217.69
G-H	£ 7.92	£ -	£ -	£ 7.86	£12.41	£17.66	£25.83	£53.00	£55.58	£36.42	£35.63	£34.14	£31.90	£32.13	£32.86	£33.08	£48.89	£50.30	£33.57	£30.00	£25.64	£20.48	£16.09	£11.54	£ 652.92
G-L	£ 5.71	£ -	£ -	£ 8.22	£12.73	£17.66	£26.27	£52.15	£57.67	£32.21	£29.36	£27.43	£25.60	£26.77	£27.78	£27.77	£40.54	£41.67	£28.21	£26.01	£22.67	£17.06	£12.98	£ 8.48	£ 574.91
G-CH	£ -	£ -	£ -	£ 1.60	£ 2.83	£ 5.22	£10.87	£28.55	£35.59	£15.91	£14.15	£12.33	£ 9.58	£ 9.69	£10.98	£12.19	£19.67	£22.90	£16.00	£12.77	£ 9.49	£ 5.64	£ 3.97	£ 2.04	£ 261.99
S-G	£ 6.65	£ -	£ -	£ 7.56	£ 9.77	£13.16	£19.15	£39.28	£45.28	£25.62	£23.66	£21.28	£18.76	£18.76	£19.93	£21.45	£33.69	£35.08	£22.78	£18.80	£15.12	£11.54	£10.33	£ 8.24	£ 445.91
S-C	£ 1.03	£ -	£ -	£ 1.18	£ 1.60	£ 2.46	£ 6.35	£22.77	£46.14	£12.30	£ 8.58	£ 6.74	£ 5.30	£ 4.55	£ 4.95	£ 5.90	£10.50	£12.29	£ 8.42	£ 7.00	£ 5.34	£ 3.73	£ 2.94	£ 2.03	£ 182.10
S-H	£ 7.99	£ -	£ -	£ 7.73	£13.22	£18.61	£26.55	£53.23	£59.78	£34.81	£34.48	£33.14	£31.07	£31.34	£32.76	£34.01	£52.91	£56.79	£38.43	£33.27	£27.29	£20.77	£16.74	£11.70	£ 676.62
S-L	£ 2.34	£ -	£ -	£ 3.57	£ 6.00	£ 8.20	£11.90	£22.80	£23.66	£12.93	£11.72	£10.91	£10.23	£10.85	£11.66	£12.14	£18.93	£20.63	£14.28	£12.66	£10.48	£ 7.31	£ 5.73	£ 3.52	£ 252.46
S-CH	£ -	£ -	£ -	£ 1.35	£ 2.48	£ 4.52	£ 9.22	£23.79	£28.75	£12.77	£11.39	£ 9.86	£ 7.62	£ 7.70	£ 8.94	£10.28	£17.43	£21.08	£14.90	£11.50	£ 8.20	£ 4.65	£ 3.39	£ 1.72	£ 221.53
C-G	£ 1.63	£ -	£ -	£ 1.86	£ 2.31	£ 3.55	£ 6.04	£13.70	£17.45	£10.45	£ 9.61	£ 8.51	£ 7.40	£ 7.24	£ 8.09	£ 9.45	£17.61	£24.26	£16.28	£11.75	£ 8.80	£ 6.72	£ 5.28	£ 4.13	£ 202.13
C-S	£ 1.03	£ -	£ -	£ 1.18	£ 1.60	£ 2.46	£ 4.15	£ 9.22	£11.28	£ 6.76	£ 6.21	£ 5.36	£ 4.55	£ 4.41	£ 5.53	£ 7.30	£15.87	£24.87	£16.93	£11.16	£ 7.67	£ 5.52	£ 4.53	£ 3.62	£ 161.22
C-H	£ 3.28	£ -	£ -	£ 2.75	£ 5.69	£ 9.20	£14.87	£32.38	£39.71	£24.43	£24.17	£23.00	£21.39	£21.24	£22.59	£24.38	£41.37	£51.51	£35.47	£29.03	£23.11	£17.59	£12.84	£ 8.65	£ 488.65
C-L	£ -	£ -	£ -	£ 0.95	£ 1.96	£ 3.10	£ 5.24	£10.92	£12.47	£ 7.29	£ 6.57	£ 6.01	£ 5.57	£ 5.79	£ 6.78	£ 7.96	£15.87	£24.54	£17.05	£12.16	£ 9.04	£ 6.50	£ 4.70	£ 3.18	£ 173.65
C-CH	£ -	£ -	£ -	£ 0.49	£ 1.00	£ 2.07	£ 4.79	£13.40	£17.58	£ 8.22	£ 7.34	£ 6.30	£ 4.83	£ 4.79	£ 5.55	£ 6.48	£11.30	£14.24	£10.29	£ 8.23	£ 5.91	£ 3.36	£ 2.16	£ 1.00	£ 139.35
H-G	£ 7.92	£ -	£ -	£ 7.86	£12.41	£17.66	£25.83	£53.00	£55.58	£36.42	£35.63	£34.14	£31.90	£32.13	£32.86	£33.08	£48.89	£50.30	£33.57	£30.00	£25.64	£20.48	£16.09	£11.54	£ 652.92
H-S	£ 7.99	£ -	£ -	£ 7.73	£13.22	£18.61	£26.55	£53.23	£59.78	£34.81	£34.48	£33.14	£31.07	£31.34	£32.76	£34.01	£52.91	£56.79	£38.43	£33.27	£27.29	£20.77	£16.74	£11.70	£ 676.62
H-C	£ 3.28	£ -	£ -	£ 2.75	£ 5.69	£ 9.20	£16.65	£43.35	£67.94	£28.90	£26.09	£24.11	£21.99	£21.36	£22.13	£23.25	£37.02	£41.33	£28.59	£25.67	£21.22	£16.14	£11.55	£ 7.37	£ 505.58
H-L	£ 4.96	£ -	£ -	£ 6.05	£12.51	£18.00	£26.26	£51.28	£55.53	£31.86	£31.07	£30.71	£29.96	£31.26	£32.15	£31.58	£46.40	£49.29	£34.35	£32.19	£27.93	£21.05	£15.16	£ 8.97	£ 628.51
H-CH	£ -	£ -	£ -	£ 0.50	£ 1.09	£ 2.07	£ 4.25	£10.98	£13.47	£ 6.15	£ 5.69	£ 5.14	£ 4.12	£ 4.19	£ 4.72	£ 5.16	£ 8.32	£ 9.91	£ 7.13	£ 5.80	£ 4.32	£ 2.55	£ 1.71	£ 0.81	£ 108.08
L-G	£ 5.71	£ -	£ -	£ 8.22	£12.73	£17.66	£26.27	£52.15	£57.67	£32.21	£29.36	£27.43	£25.60	£26.77	£27.78	£27.77	£40.54	£41.67	£28.21	£26.01	£22.67	£17.06	£12.98	£ 8.48	£ 574.91
L-S	£ 2.34	£ -	£ -	£ 3.57	£ 6.00	£ 8.20	£11.90	£22.80	£23.66	£12.93	£11.72	£10.91	£10.23	£10.85	£11.66	£12.14	£18.93	£20.63	£14.28	£12.66	£10.48	£ 7.31	£ 5.73	£ 3.52	£ 252.46
L-C	£ -	£ -	£ -	£ 0.95	£ 1.96	£ 3.10	£ 7.39	£24.21	£46.66	£12.72	£ 8.89	£ 7.36	£ 6.30	£ 5.93	£ 6.21	£ 6.58	£10.61	£12.21	£ 8.71	£ 8.08	£ 6.75	£ 4.74	£ 3.14	£ 1.62	£ 194.14
L-H	£ 4.96	£ -	£ -	£ 6.05	£12.51	£18.00	£26.26	£51.28	£55.53	£31.86	£31.07	£30.71	£29.96	£31.26	£32.15	£31.58	£46.40	£49.29	£34.35	£32.19	£27.93	£21.05	£15.16	£ 8.97	£ 628.51
L-CH	£ -	£ -	£ -	£ 0.87	£ 1.91	£ 3.56	£ 7.37	£18.57	£21.68	£ 9.49	£ 8.39	£ 7.52	£ 6.08	£ 6.36	£ 7.23	£ 7.83	£12.52	£14.95	£10.84	£ 9.06	£ 6.86	£ 3.88	£ 2.52	£ 1.08	£ 168.59
CH-G	£ -	£ -	£ -	£ 1.60	£ 2.83	£ 5.22	£10.87	£28.55	£35.59	£15.91	£14.15	£12.33	£ 9.58	£ 9.69	£10.98	£12.19	£19.67	£22.90	£16.00	£12.77	£ 9.49	£ 5.64	£ 3.97	£ 2.04	£ 261.99
CH-S	£ -	£ -	£ -	£ 1.35	£ 2.48	£ 4.52	£ 9.22	£23.79	£28.75	£12.77	£11.39	£ 9.86	£ 7.62	£ 7.70	£ 8.94	£10.28	£17.43	£21.08	£14.90	£11.50	£ 8.20	£ 4.65	£ 3.39	£ 1.72	£ 221.53
CH-C	£ -	£ -	£ -	£ 0.49	£ 1.00	£ 2.07	£ 4.79	£13.40	£17.58	£ 8.22	£ 7.34	£ 6.30	£ 4.83	£ 4.79	£ 5.55	£ 6.48	£11.30	£14.24	£10.29	£ 8.23	£ 5.91	£ 3.36	£ 2.16	£ 1.00	£ 139.35
CH-H	£ -	£ -	£ -	£ 0.50	£ 1.09	£ 2.07	£ 4.25	£10.98	£13.47	£ 6.15	£ 5.69	£ 5.14	£ 4.12	£ 4.19	£ 4.72	£ 5.16	£ 8.32	£ 9.91	£ 7.13	£ 5.80	£ 4.32	£ 2.55	£ 1.71	£ 0.81	£ 108.08
CH-L	£ -	£ -	£ -	£ 0.87	£ 1.91	£ 3.56	£ 7.37	£18.57	£21.68	£ 9.49	£ 8.39	£ 7.52	£ 6.08	£ 6.36	£ 7.23	£ 7.83	£12.52	£14.95	£10.84	£ 9.06	£ 6.86	£ 3.88	£ 2.52	£ 1.08	£ 168.59
																								Sum For Day	£ 10,387
																								Sum For Year (Millions)	£ 3,793.8

Unweighted PPK = 30p

These figures can be multiplied by the congestion data and summed to find the total sales per day, and then by multiplying, the sales per year. This is shown in the table above.

ESTIMATION OF PERCENTAGE OF TRAVELLERS UNDER 16

We can calculate the proportion of passengers under 16 by using a passenger survey report across UK airports [111].

It gives the following figures:

AIRPORT	TOTAL DAILY PASSENGERS	NUMBER OF UNDER 16'S	PERCENTAGE OF UNDER 16'S
GATWICK	51600	2373.6	4.60%
STANSTED	35040	525.6	1.50%
CITY	26640	106.56	0.40%
HEATHROW	69840	1746	2.50%
LUTON	42960	3823.44	8.90%
OVERALL	**226080**	**8575.2**	**3.79%**

We can also use government transport data of train journeys per person per age band per year from [111], and combine this with UK demographic data from [112] to estimate the same percentage for trains in general:

TRIPS /PERSON /AGE BAND /YEAR		**PERCENTAGE OF TOTAL POPULATION**		
AGE	**Trips**	**In this age range**	**Weighted Trips**	**Percentage of Trips**
0-16	8	19.6	156.8	**4.71%**
17-20	38	4.4	167.2	
21-29	60	13.6	816	
30-39	64	13	832	
40-49	36	14.6	525.6	
50-59	46	12.2	561.2	
60-69	16	10.8	172.8	
70+	8	11.8	94.4	
TOTAL		100	3326	

We can calculate from our congestion data that 43% of travellers are commuters. We can use this to find a weighted average of the percentages of travellers under 16, which is

$$0.43\times 4.71\% + 0.57\times 3.79\% = 4.19\%$$

We can also estimate the percentage of commuters that will buy season tickets. This is given by [110], and we can average it over the last 6 years to find a reliable percentage:

TICKET SALES (MILLIONS)				
YEAR	Ordinary	Season	Total	Season %
2010-11	757.0	596.8	1353.8	44%
2011-12	833.9	626.1	1460.0	43%
2012-13	870.8	630.1	1500.9	42%
2013-14	895.6	690.8	1586.4	44%
2014-15	951.8	701.9	1653.7	42%
2015-16	1003.7	711.6	1715.3	41%
			Average	43%

We estimate an additional 5% of air passengers hold season cards.

Finally, we can generously estimate that 70% of commuters purchase return tickets, or 30.1% of total passengers. This gives an overall sales reduction of 15.76%:

PERCENTAGE OF TRAVELLERS UNDER 15	4.19%
CHILD DISCOUNT	40.00%
REMAINING	98.32%
PERCENTAGE OF PEOPLE WITH SEASON TICKET	26.50%
SEASON DISCOUNT (RAIL CARD)	33.33%
REMAINING	89.64%
PERCENTAGE OF TICKETS THAT ARE RETURNS	30.10%
RETURN TICKET DISCOUNT	20.00%
FINAL REMAINING	84.24%

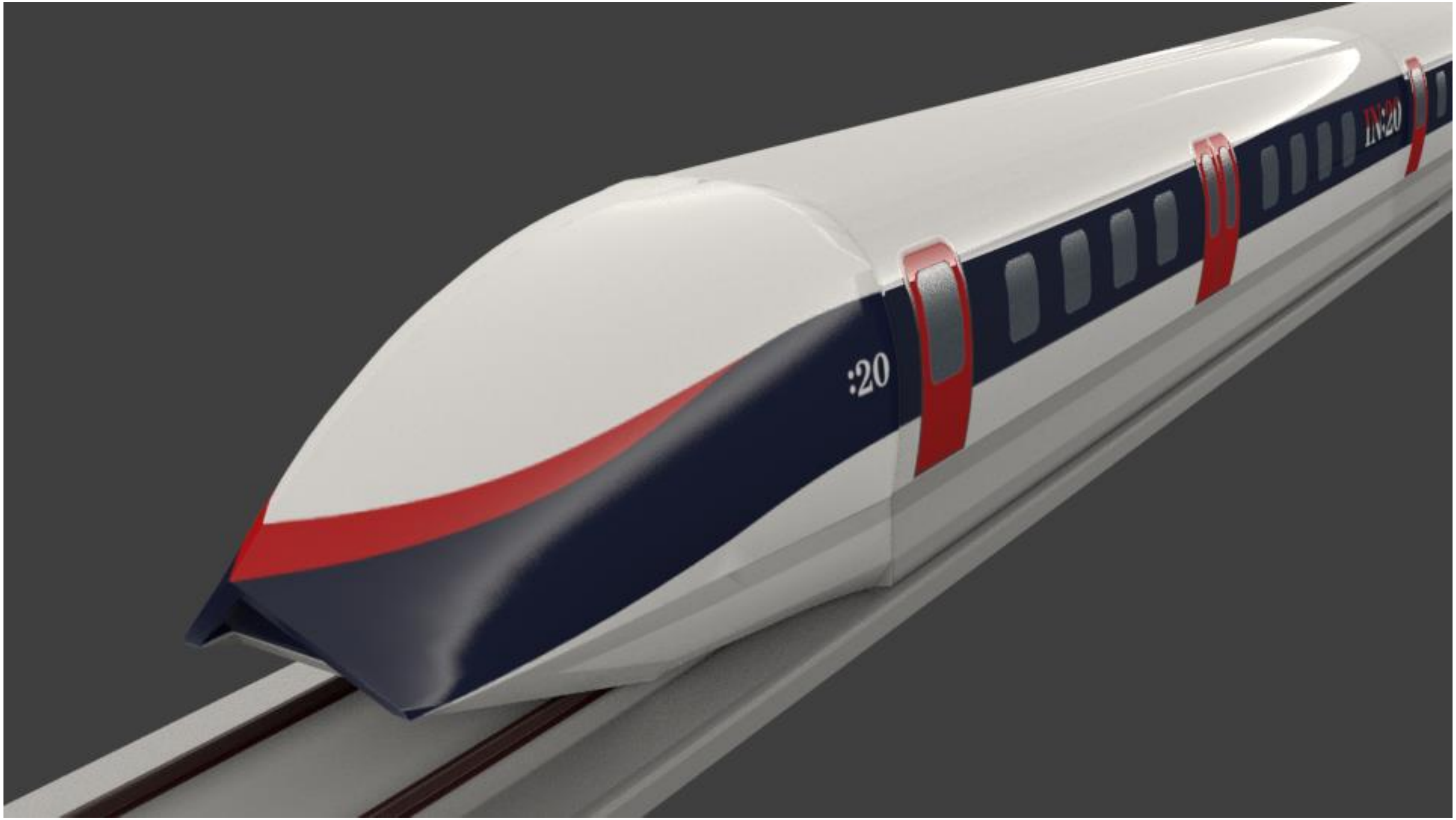
15. FUTURE FINANCE

Having devised a timetable and assigned costs to various subsections of the project, we were able to find the cost distribution across the project's construction period, calculating costs every quarter of a year. Sales could also be calculated.

- Route segment and station construction costs were distributed evenly across their construction periods.
- Energy & maintenance costs were applied every quarter from when a track segment is opened. The proportion of the cost corresponding to a given route was found by calculating the percentage of the total, summed route distances that the particular route contains.
- Costs are assumed to increase with inflation.
- Ticket prices also increase with inflation, to stay competitive.
- Temporary lines earn a quarter as much as fully-operational lines.

The following table is attained for the first 4 years:

YEAR	QUARTER	BALANCE	CASH FLOW	COSTS	INCOME	REASON
1	1	-£83.73	-£83.73	-£333.73	£250	Independent Donations
	2	-£417.45	-£333.73	-£333.73		
	3	-£751.18	-£333.73	-£333.73		
	4	-£1,084.90	-£333.73	-£333.73		
2	1	-£1,562.65	-£477.74	-£477.74		
	2	-£2,040.39	-£477.74	-£477.74		
	3	-£2,518.14	-£477.74	-£477.74		
	4	-£2,959.03	-£440.90	-£440.90		
3	1	-£4,136.41	-£1,177.38	-£1,191.47	£13.54	Sales
	2	-£5,313.79	-£1,177.38	-£1,191.47	£13.54	Sales
	3	-£6,401.08	-£1,087.28	-£1,194.18	£102.75	Sales
	4	-£7,420.33	-£1,019.25	-£1,126.15	£102.75	Sales
4	1	-£8,667.53	-£1,247.20	-£1,356.24	£102.75	Sales
	2	-£9,914.73	-£1,247.20	-£1,356.24	£102.75	Sales
	3	-£11,057.60	-£1,142.87	-£1,251.91	£102.75	Sales
	4	-£12,329.46	-£1,271.86	-£1,380.90	£102.75	Sales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